Does Scrambling of Q.Information Imply Q.Chaos?

Ali Mollabashi

پژوهشگاه دانش‌های بنیادی

Based on 2408.12089 + upcomming work
in collaboration with **Saleh Rahimi-Keshari**

Workshop on Dynamics and Scrambling of Quantum Information

IPM, 28 Azar 1403

Introduction to Scrambling of QI

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- Haar-Scrambled states: any state evolved by a Haar random unitary

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- Q. Integrability: a set of N commuting operators A_i such that: $[H, A_i] = 0$

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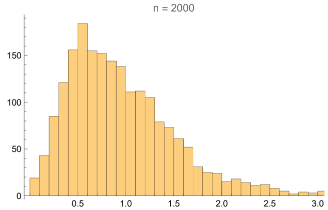
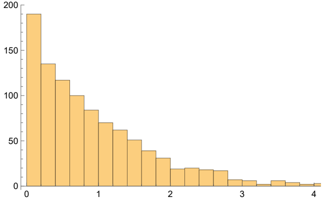
- The most widely accepted definition is the level statistics
Chaotic theory: Wigner-Dyson statistics $P(s) = s^\alpha \, e^{-s}$





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What happens at the fusion point of **randomness** and **Q. integrability**?

Do we expect **scrambling**?

Our Hamiltonian Setup

Generic quadratic bosonic model

$$H = \frac{1}{2} \sum_{n=1}^{N} \left[p_i \mathbb{P}_{ij} p_j + q_i \mathbb{Q}_{ij} q_j + q_i \mathbb{R}_{ij} p_j \right]$$

A canonical transformation: $H = \frac{1}{2} \sum_{n=1}^{N} \Omega_n a_n^{\dagger} a_n$
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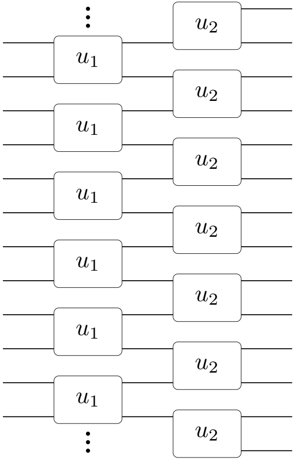
2. Passive Random Mixing ($\mathbb{Q} \in$ GOE, GUE)

Our setup: Quantum Circuits



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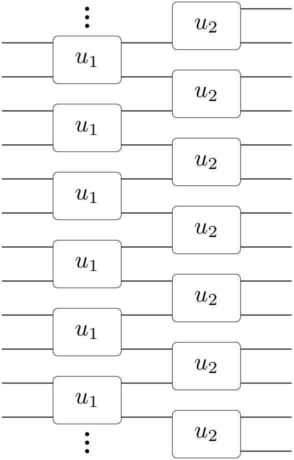


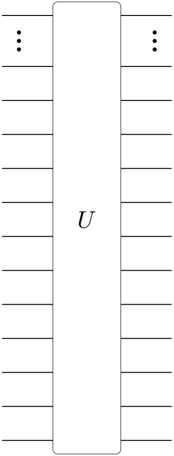
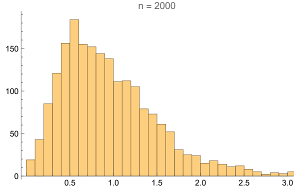


Figure: Local Unitary (a single time step) — Completely Random Unitary

Chaos versus Single-particle Chaos

- Chaos: Wigner-Dyson statistics $P(s) = s^{\alpha}\, e^{-s}$



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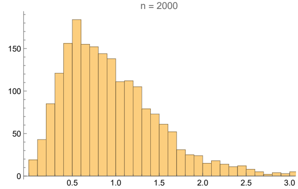
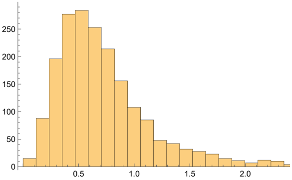


- Single-particle chaos: (Poisson statistics $P(s) = e^{-s}$)



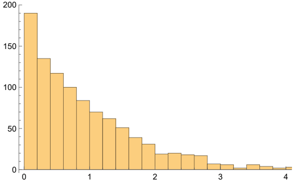


Figure: single-particle sector complete spectrum

Entanglement Measures

- For pure states, von Neumann entropy

$$\rho_A = \mathrm{Tr}_B\left[\rho_{AB}\right] \quad , \quad S_A = -\mathrm{Tr}_A\left[\rho_A \log \rho_A\right]$$

- For mixed states, Logarithmic Negativity

$$\mathcal{E} = \log \sqrt{\rho_{AB}^{\Gamma_B}{}^{\dagger} \rho_{AB}^{\Gamma_B}}$$

$$\rho_{AB} = \sum_{a_1, a_2, b_1, b_2} c_{a_1, a_2}^{b_1, b_2} \left|a_1\right\rangle \left\langle a_2\right| \otimes \left|b_1\right\rangle \left\langle b_2\right|$$

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- *Tripartite Mutual Information (TMI)*

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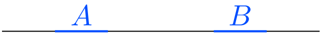
Entanglement Spread

$S_{A\cup B}$: A Measure for Information Scrambling



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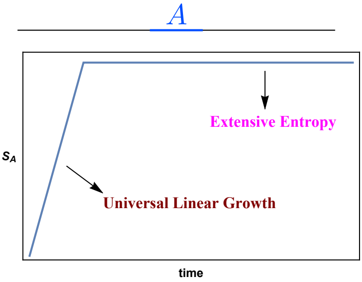

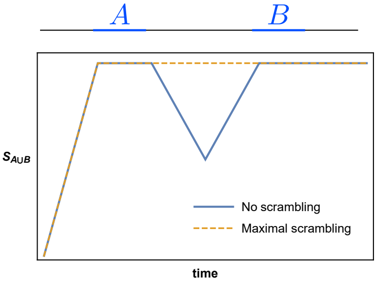

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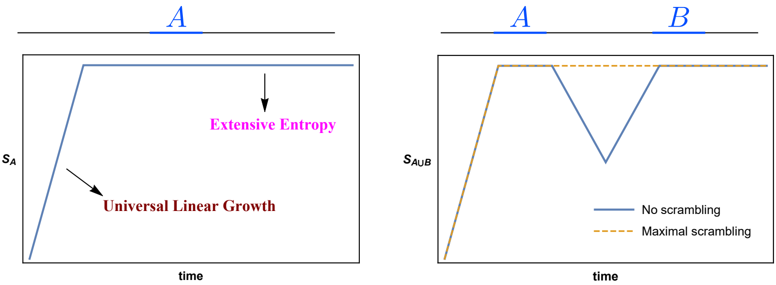

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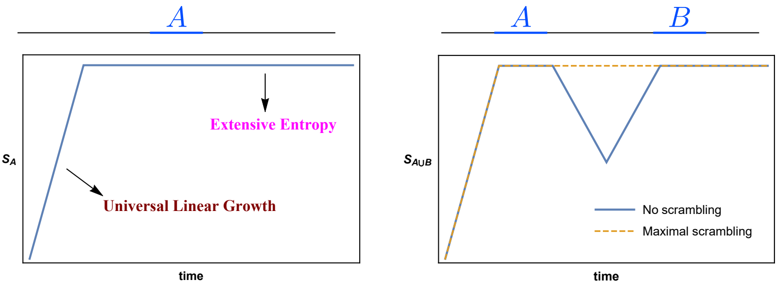

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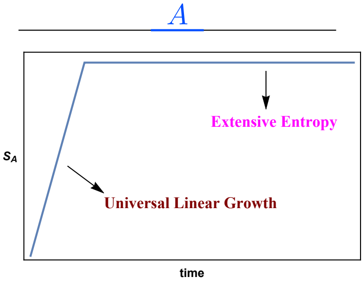

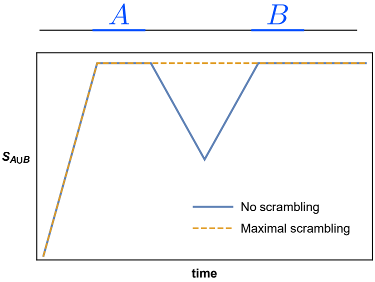

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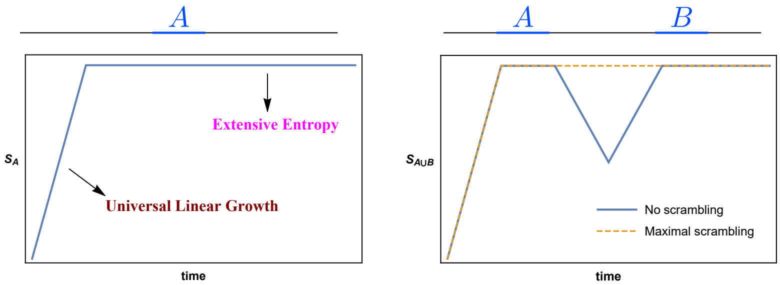

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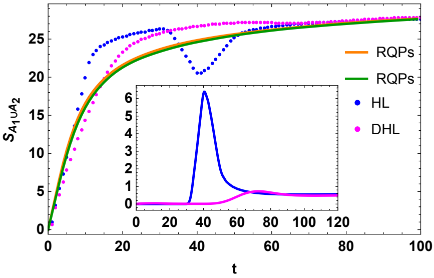
- Dip in $S_{A\cup B}$ $\Leftrightarrow$ Peak in MI

Entanglement Spread: Disordered Local Model

- Entanglement spread due to a quantum quench



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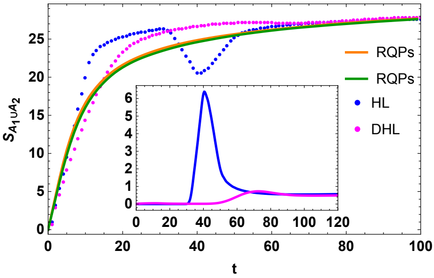

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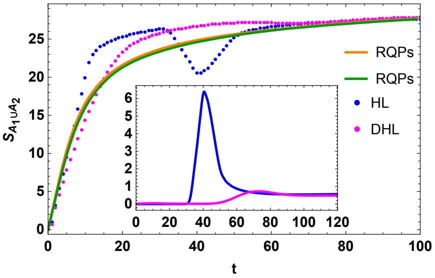
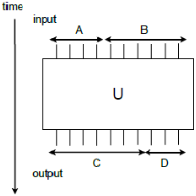


- delocalization in mutual information
- Physical Description: Randomly distribution for quasi-particles **group velocity** and **entropy density**

Tripartite Mutual Information (TMI)

- Strong scrambler: local measurements on C or D cannot reveal much about local disturbence in A but $I(A:CD)$ is the total amount of information we can learn about A



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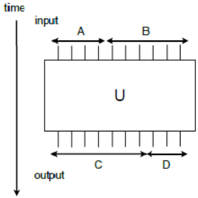

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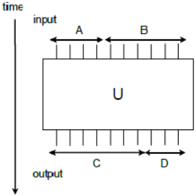

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- Tripartite Mutual Information:

$$\begin{aligned} I_3(A:C:D) &= S_A + S_C + S_D - S_{AC} - S_{AD} - S_{CD} + S_{ACD} \\ &= I(A:C) + I(A:D) - I(A:CD) \end{aligned}$$

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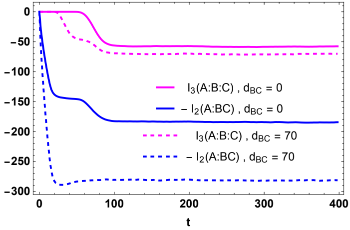

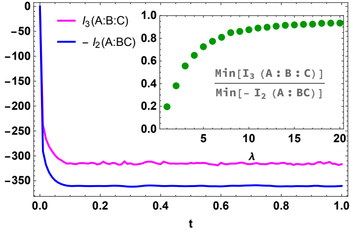

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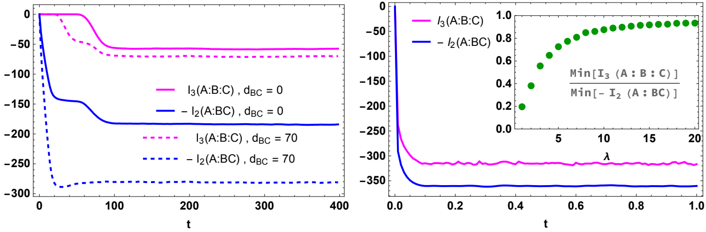

- Evolution of a passive Haar unitary in large squeezing limit denotes scrambling

SFF

- The analytical continuation of the partition function

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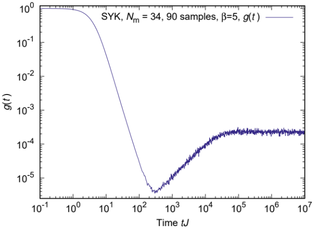
- Slope-dip-ramp-plateau picture



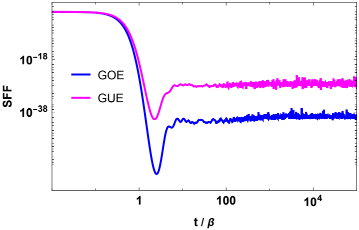


Figure: LEFT: Borrowed from Cotler et al. JHEP 1705:118, 2017

SFF

- The analytical continuation of the partition function

$$Z(t,\beta) = \mathrm{Tr}\left(e^{-(\beta+it)H}\right)$$

- Spectral form factor

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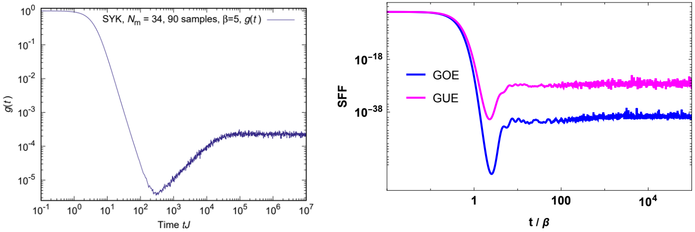

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- SYK_2: exponential ramp [Winer-Jian-Swingle '20]

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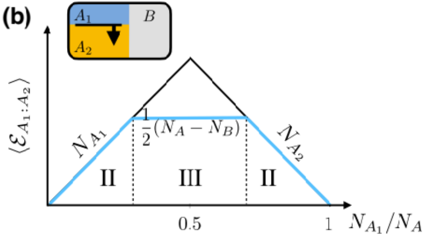

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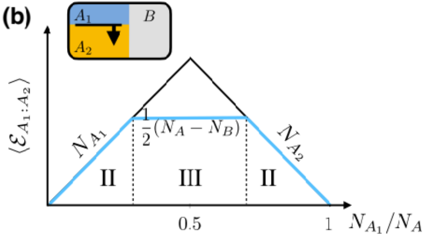

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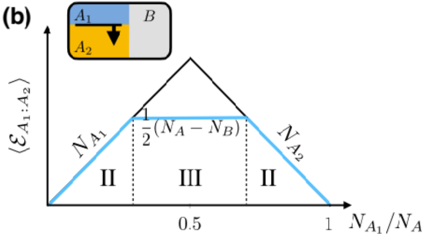

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- $L_A \gg L_B$: Page state limit
- $L_A \ll L_B$: A is maximally entangled with B

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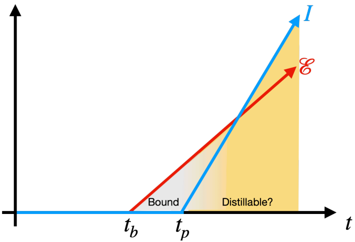

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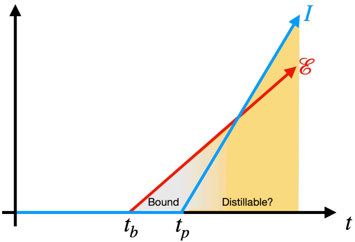

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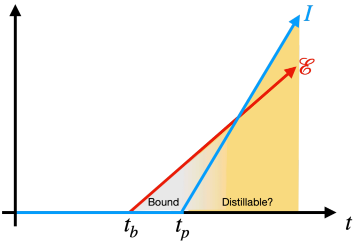

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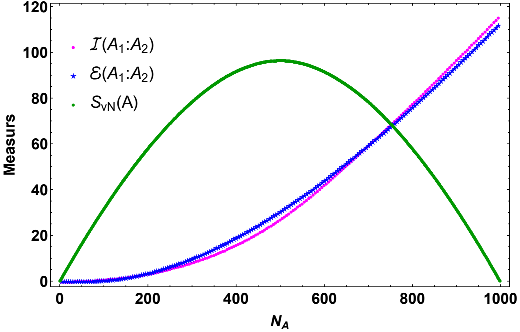

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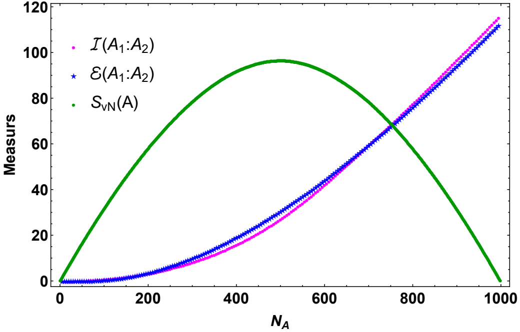

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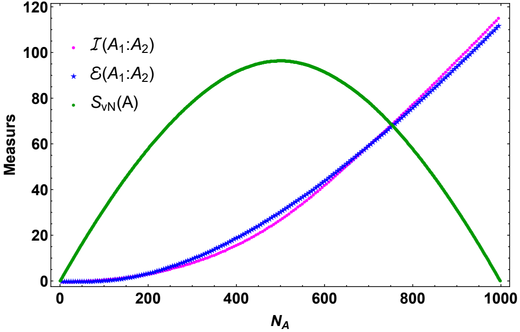

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- Logarithmic negativity is *continuous* for this family of states [Eisert-Simon-Plenio '01]

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  2. Discrete sine-Gordon theory [Xu-Scaffidi-Cao '20]
  3. LMG spin model [Pilatowsky-Cameo et al. '20]