1st IPM Workshop on Accelerator Physics and Engineering:

Fundamentals of Particle Accelerators

Design and Construction

Iranian Light Source Facility

Theory and Design of Charged Particle Beams

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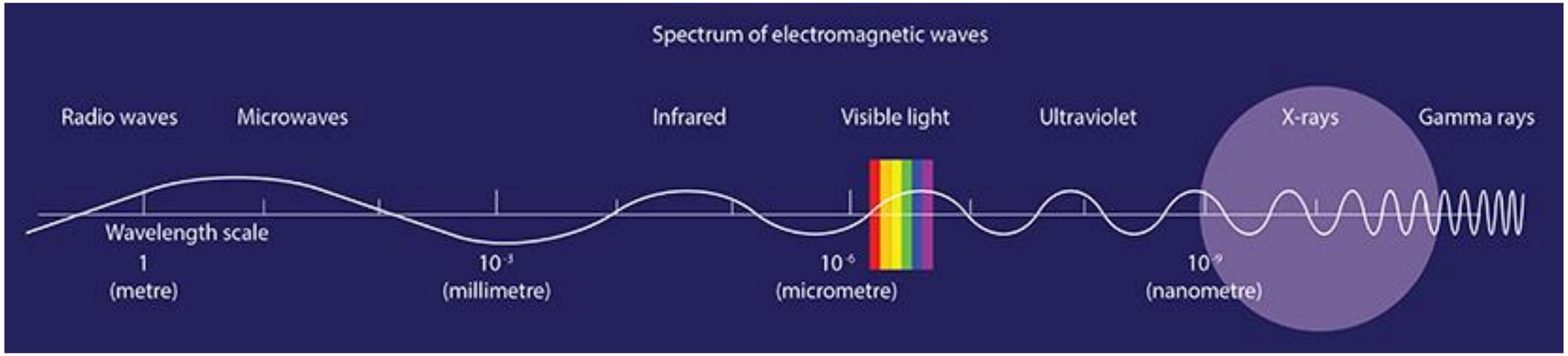
OUTLINE

- Principle of synchrotron radiation
- Synchrotron light source
- Basic physics of storage ring
- Emittance and lattice structure
- Lattice structure for ILSF storage ring
- Nonlinear optimization

Properties of Radiation

- Spectrum of electro-magnetic radiation



Synchrotron radiation is used for experiments typically over this region

Properties of Radiation

Light characteristics suitable for experiments

- High brilliance
- Coherence
- Polarization
- Short pulse
- Stability
- Wide spectral range
- Higher photon energies

Flux of radiation:
number of photons per second

Brightness of radiation:
flux at a specific wavelength divided by source size and divergence

Coherence of radiation:
are the photons produced in phase or not?

Polarization of radiation:
directionality of radiation field linear, circular partial/full polarization

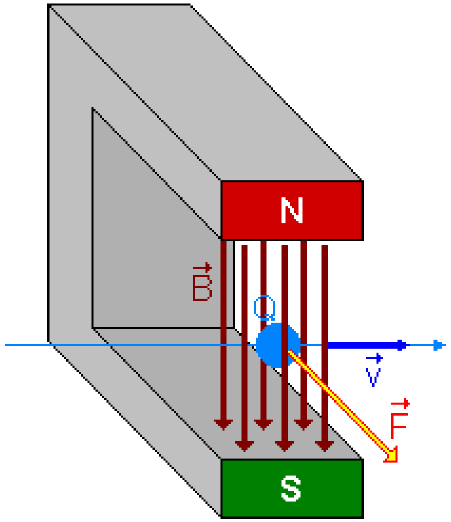
Low brightness: low photon density on sample

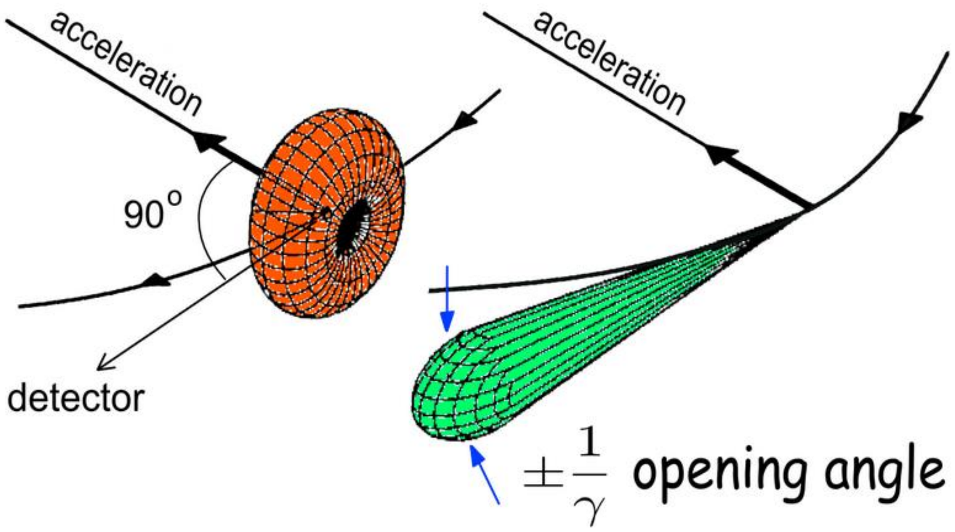
High brightness: high photon density on sample

Synchrotron light sources give some control over all these properties

Where X-RAYS Come From

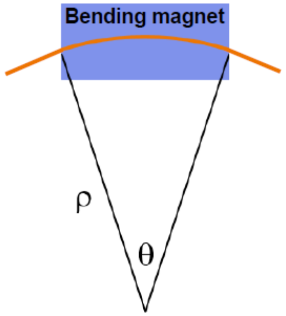
- Accelerated charged particles are emitting electromagnetic radiation.
- We can move charged particles using the Lorentz force.
  - $\vec{F} = q\,(\vec{E} + \vec{v} \times \vec{B})$
- If the charged particle is moving very fast, the radiation is emitted in a cone, with aperture $\frac{1}{\gamma}$
  - $(\gamma = \frac{E}{E_0}, E_{e0} = 0.511\ MeV, E_{p0} = 938\ MeV)$



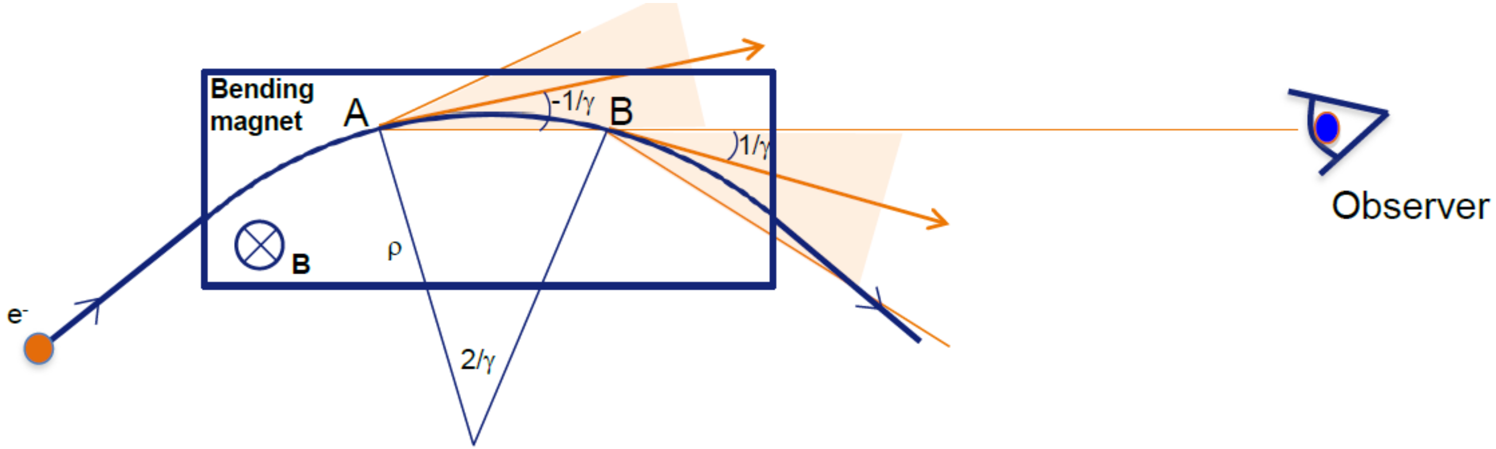


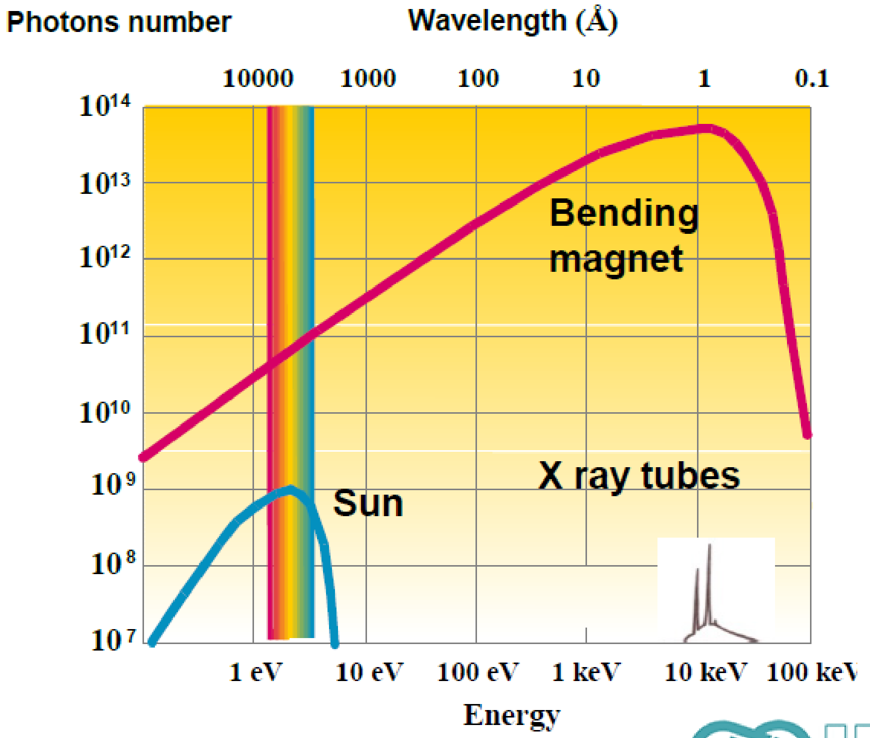
Bending Magnet Radiation

Power emitted by an ultra-relativistic particle in a bending magnet : $P = \frac{1}{6}\frac{e^2c}{\pi\epsilon_0}\frac{1}{\rho^2}\left(\frac{E}{mc^2}\right)^4$



Critical photon energy: $\varepsilon_c = \frac{3}{2}\frac{\hbar c}{\rho}\gamma^3$





What would happen if we put protons instead of electrons?

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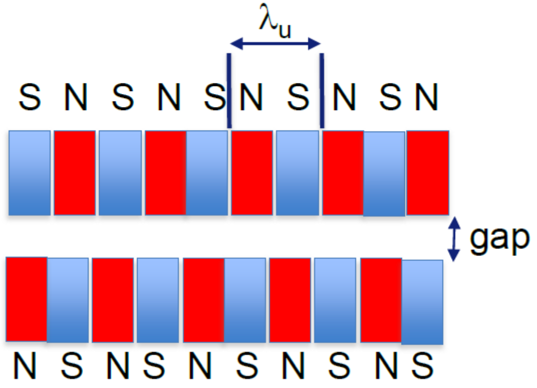
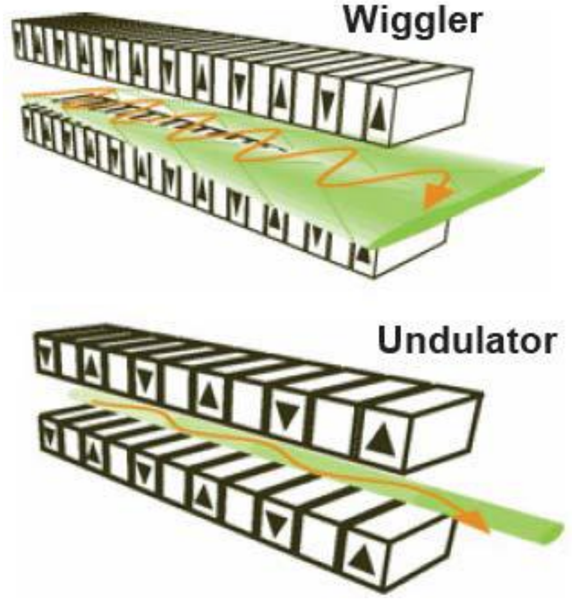
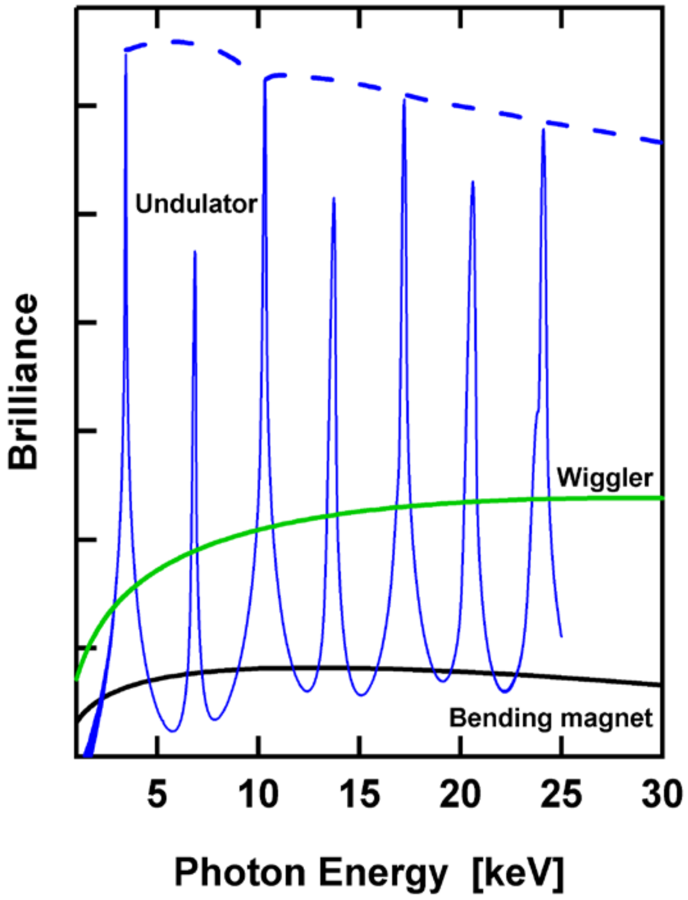
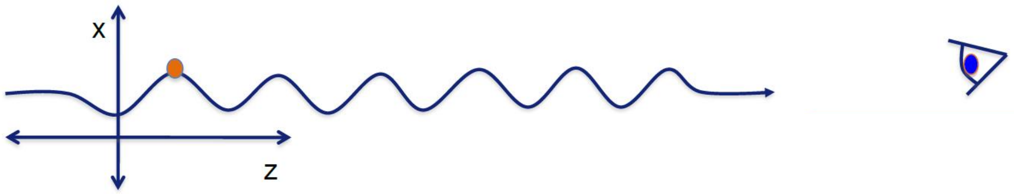
Undulator Radiation

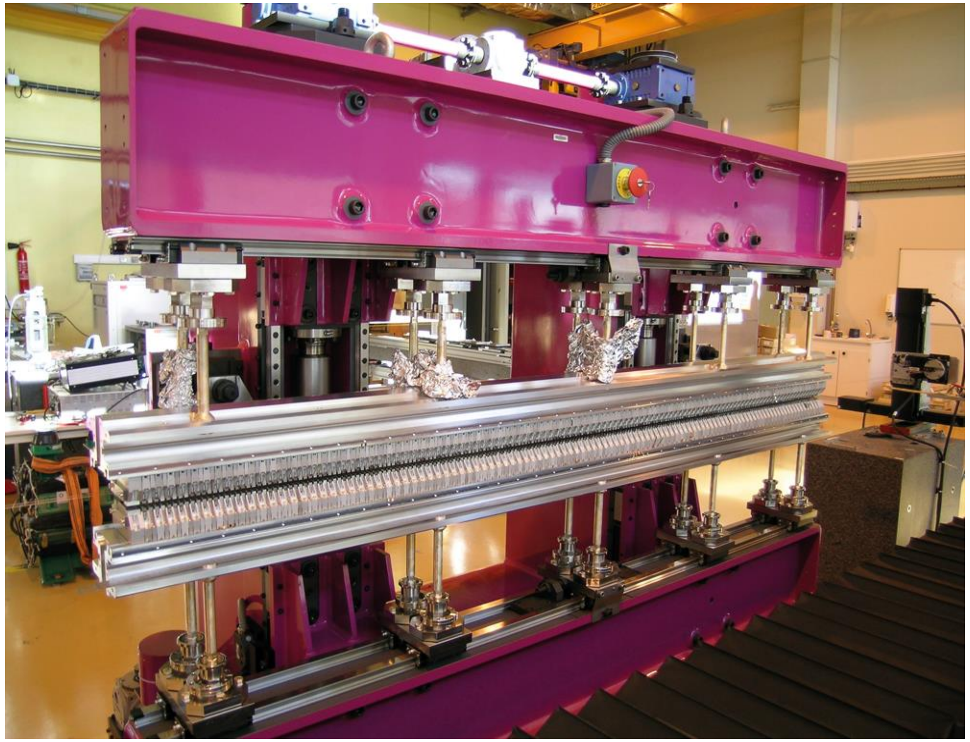
We can build magnets to have this kind of trajectory inside: undulators and wigglers



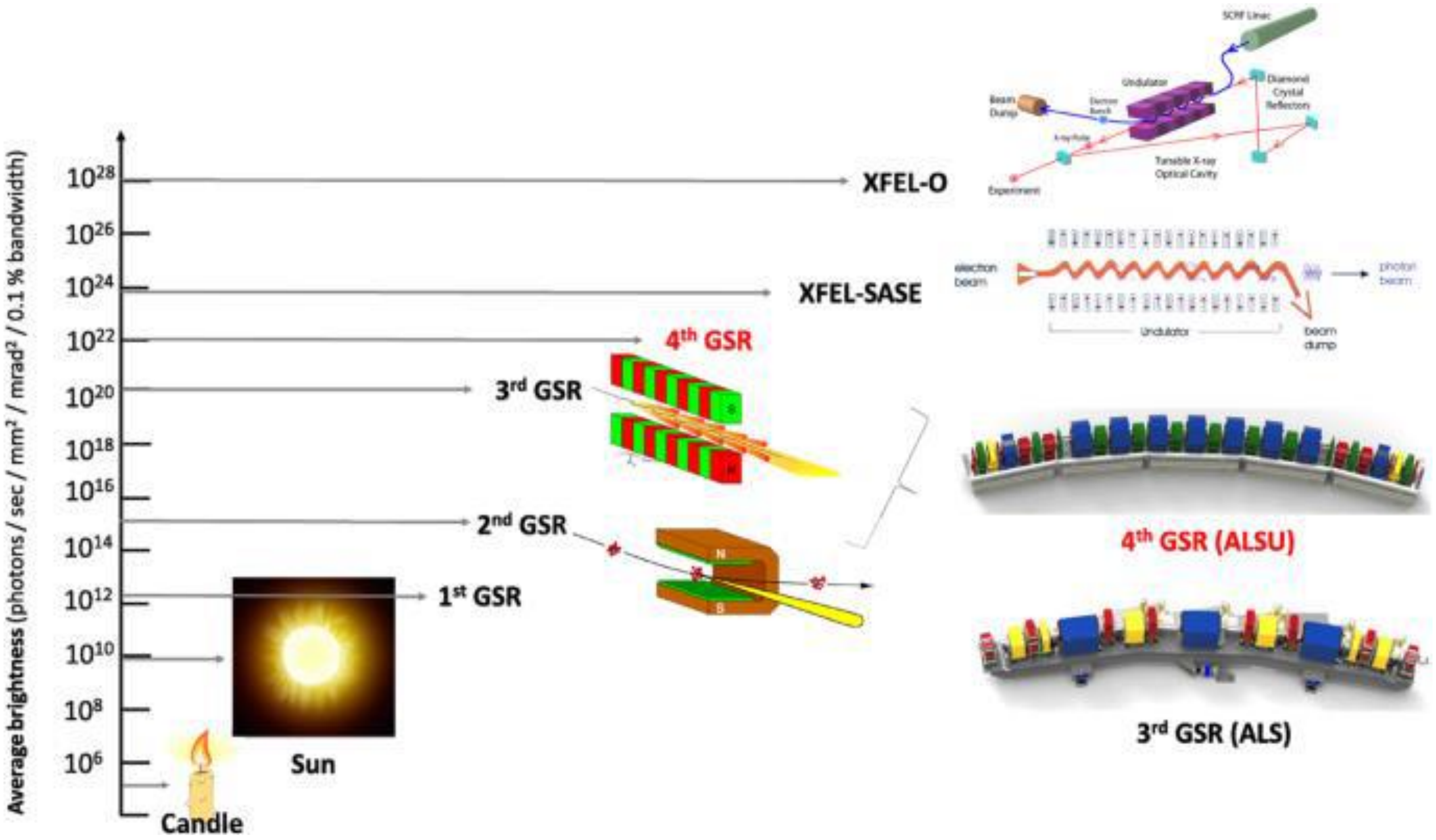




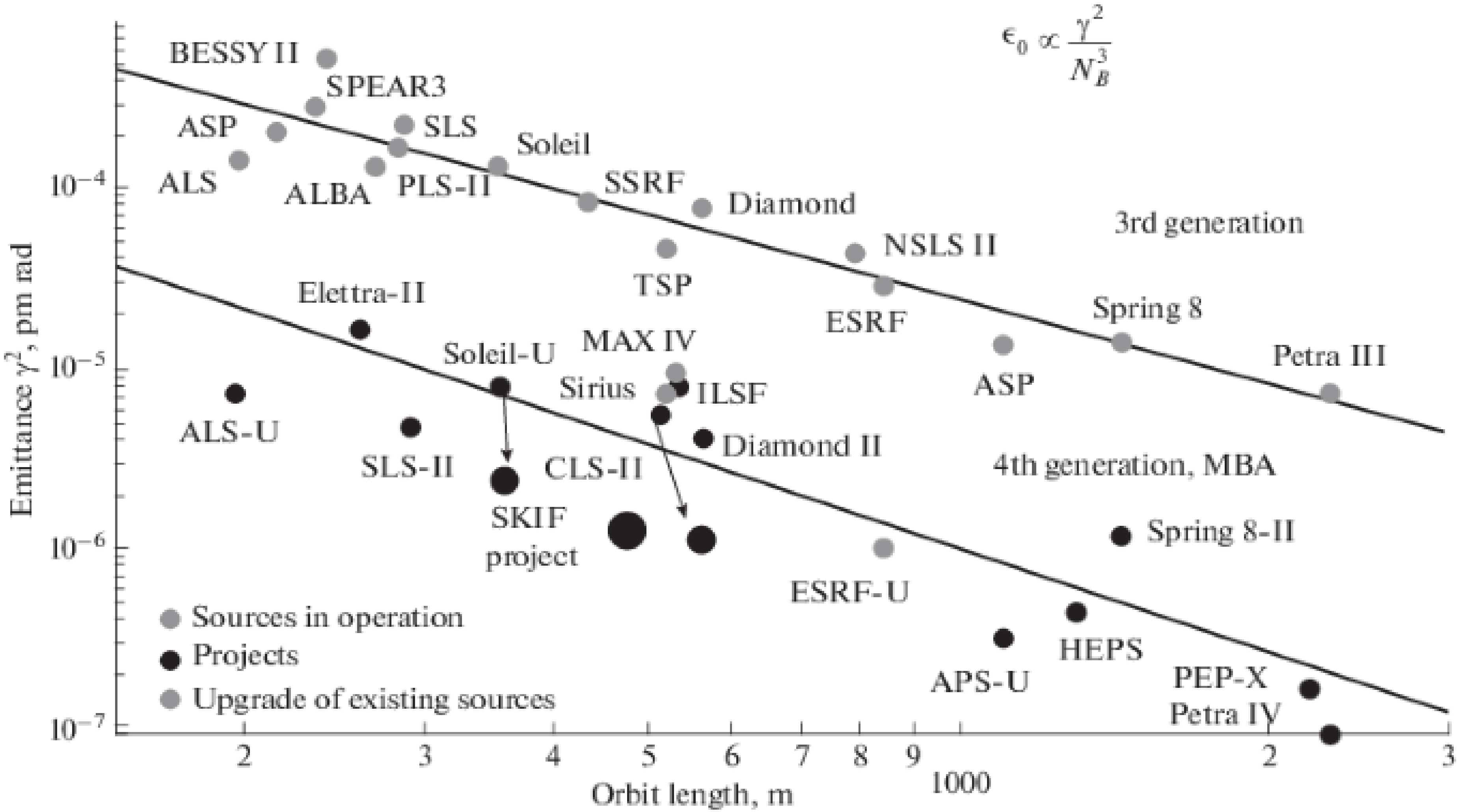




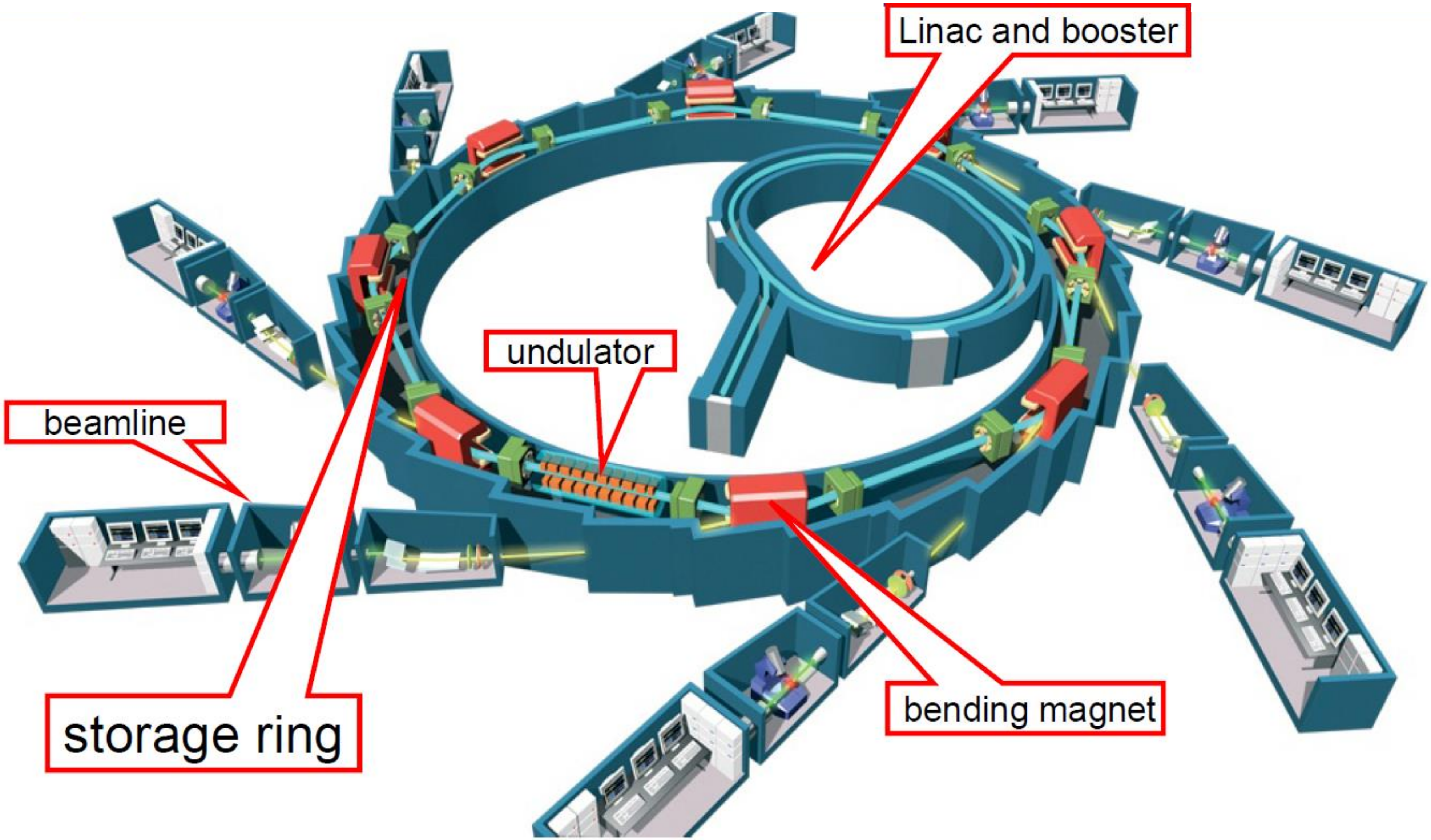
Synchrotron Light Source



Light Sources Around The World



Synchrotron Light Source



Brightness:

The concentration of radiation is called the brightness, measured in: **Photons/(s,mm², mrad²,0.1% bandwidth)**

$$B = \frac{F}{4\pi^2 \Sigma_x \Sigma_{x'} \Sigma_y \Sigma_{y'}}$$

$$\Sigma_{x,y} = \sqrt{\sigma_{x,y}^2 + \sigma_\lambda^2}$$ Convoluted size

$$\Sigma_{x',y'} = \sqrt{\sigma_{x,y}'^2 + \sigma_\lambda'^2}$$ Convoluted divergence

For Undulator with $L_u = N_u \lambda_u$

$$\sigma_\lambda = \sqrt{\frac{\lambda_n L_u}{8\pi^2}}, \sigma_\lambda' = \sqrt{\frac{\lambda_n}{2L_u}}$$

$$\lambda_n = \frac{\lambda_u}{2n\gamma^2}\left(1 + \frac{K^2}{2}\right), K = 0.9336 B_0 \lambda_u$$

Emittance:

The emittance represents the electron beam transverse size and divergence. It could be defined as a total phase space area occupied by the beam. $\boldsymbol{\epsilon_x = \sigma_x \sigma_x'}$ $\boldsymbol{\epsilon_y = \sigma_y \sigma_y'}$

$$B_n \propto \frac{F}{4\pi^2 K \boldsymbol{\epsilon_x^2}}$$ → pushing the emittance to lower values is an efficient way to increase the brightness

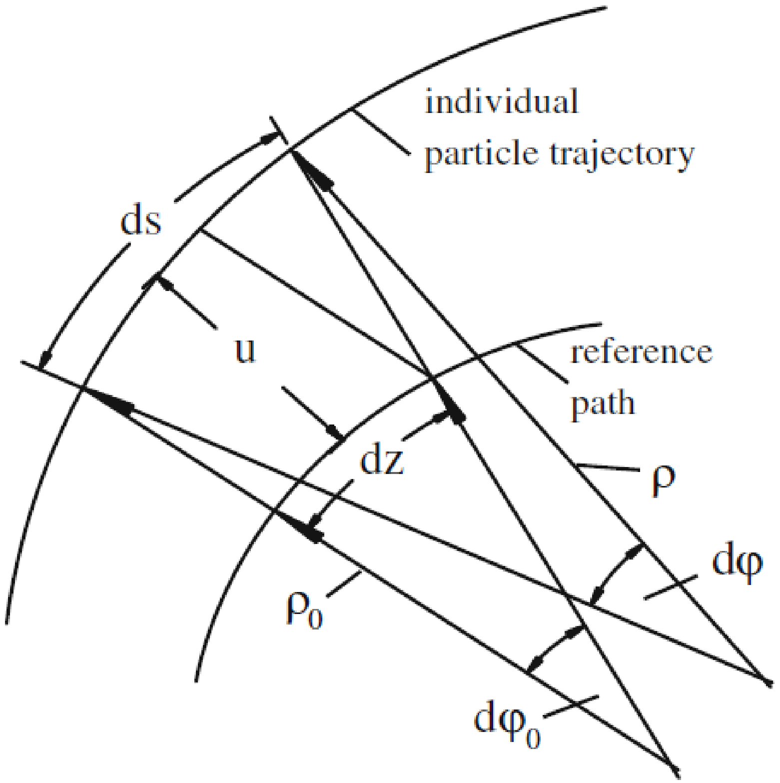
Diffraction Limit:

- Because of diffraction, the lower limit on the photon beam emittance is given approximately by the wavelength, λ. Using standard deviation values for Gaussian distributions, this diffraction-limited photon beam emittance is given by $\boldsymbol{\lambda/4\pi}$.
- For the light produced by electron beam, photon beam brightness increases as electron beam emittance decrease until the electron beam emittance reaches the diffraction limit.
- All storage rings are diffraction limited for some λ.

	Wavelength [nm]	Photon energy [eV]	Diffraction Limit [nm rad]
Visible light	**400-700**	**1.7 - 3**	**30 - 50**
UV	**10 - 400**	**3 - 123**	**0.79 - 30**
VUV	**100-200**	**6 - 12**	**8 - 16**
Soft X-ray	**1.2 - 12**	**100 – 1000**	**0.08 – 0.8**
Hard X-ray	**0.12 – 0.24**	**5000 - 10000**	**0.010 – 0.020**

Light Source Generation	Emittance [nm – rad]
Second	few hundred
Third	5-20
Fourth	<1

The most convenient coordinate system to describe particle motion is the curvilinear (Frenet-Serret) system that follows with the particle along the reference path.



Equation of motion :

$$\frac{1}{\rho}\,(\mathrm{m}^{-1}) = \frac{B}{B\rho} = 0.2998\frac{|B\,(\mathrm{T})\,|}{\beta\,E\,(\mathrm{GeV})} \qquad \kappa_x = \frac{1}{1+\delta}\left(\kappa_{0x} + kx + \tfrac{1}{2}mx^2 + \ldots\right),$$

$$\frac{1}{p} = \frac{1}{p_0(1+\delta)} \approx \frac{1}{p_0}(1-\delta+\ldots)\,.$$

$$x'' + (k+\kappa_{0x}^2)x = \kappa_{0x}(\delta-\delta^2) + (k+\kappa_{0x}^2)x\delta - \tfrac{1}{2}mx^2 - \kappa_0 kx^2 + \mathcal{O}(3).$$

- Focusing term: $(k+\kappa_{0x}^2)$
- Dispersive term: $\kappa_{0x}(\delta-\delta^2)$
- Source of Chromatic error: $(k+\kappa_{0x}^2)x\delta$
- Sextupole term: $-\tfrac{1}{2}mx^2$

equations of motion in the approximation of linear beam dynamics :

$$x'' + \left(k_0 + \kappa_{0x}^2\right) x = 0\,,$$

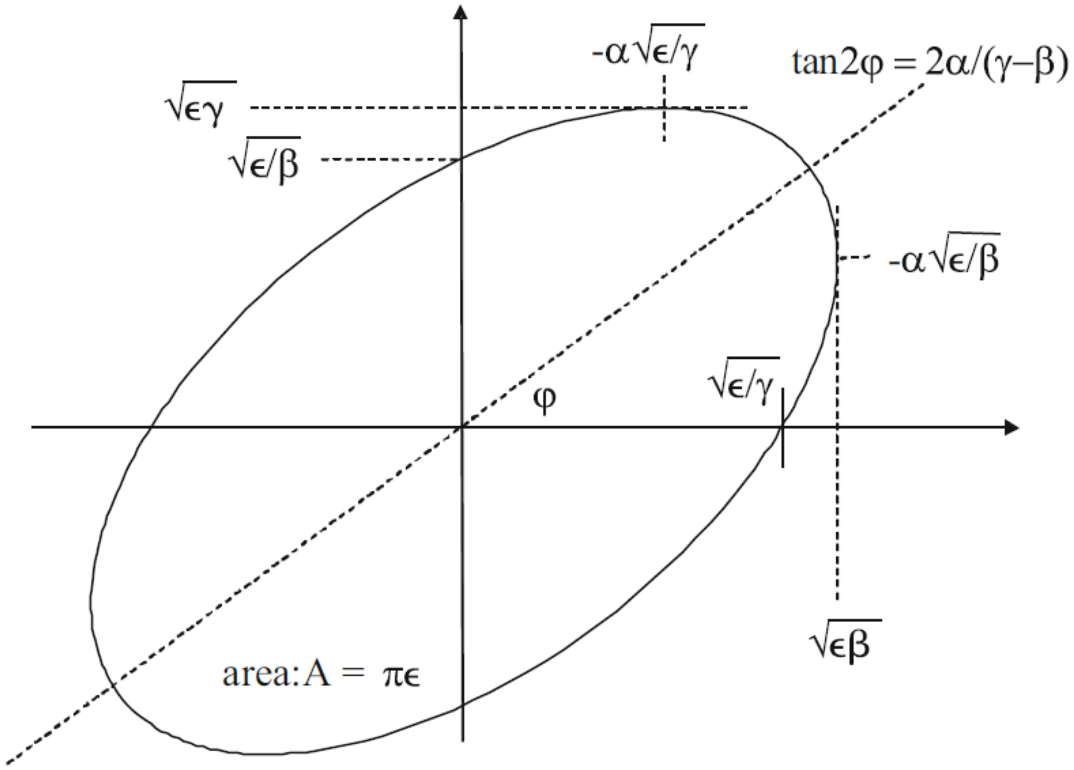
$$y'' - k_0 y = 0\,.$$

$$u'' + k(z)\, u = 0,$$

$$u(z) = \sqrt{\epsilon}\sqrt{\beta(z)}\cos[\psi(z) - \psi_0],$$

$$\psi(z) = \int_0^z \frac{\mathrm{d}\bar{z}}{\beta(\bar{z})} + \psi_0.$$

$$\gamma u^2 + 2\alpha\, uu' + \beta\, u'^2 = \epsilon.$$



Low Emittance Lattices and the Diffraction Limit

The horizontal emittance in an electron storage ring scales as the square of the electron energy and the third power of the bending angle.

$$\epsilon_{diff}(\lambda) = \frac{\lambda}{4\pi}$$

$$\epsilon_x \sim F(lattice)\ E^2\theta^3$$

$$\epsilon_x = C_q \frac{\gamma^2}{J_x} \frac{\oint H(s)/\rho(s)^3 ds}{\oint 1/\rho(s)^2\, ds}$$

$$H(s) = \gamma(s)\eta(s)^2 + 2\alpha(s)\eta(s)\eta'(s) + \beta(s)\eta'(s)^2$$

$$\gamma(s) = \frac{1+\alpha(s)^2}{\beta(s)}$$

$$\alpha(s) = -\frac{\beta'(s)}{2}$$

$$\eta(s) = \frac{\Delta \mathrm{X}}{\Delta \mathrm{p}/p_0}$$

Radiation Integrals

Many of the critical properties of the stored beam in an electron storage ring are determined by the well-known radiation integrals.
The equilibrium state, by which the equilibrium parameters such as energy spread, emittance, and bunch length are determined, is reached when quantum excitation and damping are of equal strength.

Radiation Integrals	Parameters name	Parameters
$I_1 = \oint \frac{\eta_x}{\rho} ds$	Momentum compaction	$\alpha_c = \frac{I_1}{C}$
$I_2 = \oint \frac{1}{\rho^2} ds$	Energy loss per turn	$U_0 = \frac{2r_e E^4 I_2}{3(mc^2)^3}$
$I_3 = \oint \frac{1}{\lvert\rho^3\rvert} ds$	Energy spread	$\sigma_\epsilon^2 = \frac{55}{32\sqrt{3}} \frac{\hbar\gamma^2}{mc} \frac{I_3}{2I_2+I_4}$
$I_4 = \oint \frac{\eta_x}{\rho}\left(\frac{1}{\rho^2} + 2k\right) ds$	Damping partitions	$J_x = 1 - \frac{I_4}{I_2}$, $I_\epsilon = 2 + \frac{I_4}{I_2}$
$I_5 = \oint \frac{H_x}{\lvert\rho^3\rvert} ds$	Damping time	$\tau_i = \frac{C\rho}{13.2 J_i E^3}$
$k = \frac{e}{P_0}\frac{\partial B_y}{\partial x}$ $H_x = \gamma_x \eta_x^2 + 2\alpha_x \eta_x \eta_{px} + \beta_x \eta_{px}^2$	Emittance	$\epsilon = \frac{55}{32\sqrt{3}} \frac{\hbar\gamma^2}{mc} \frac{I_5}{I_2 - I_4}$

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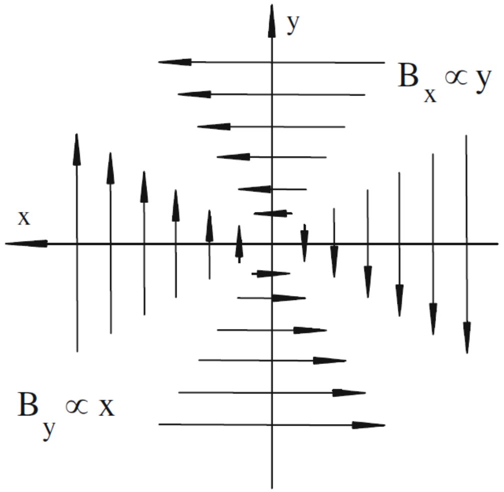
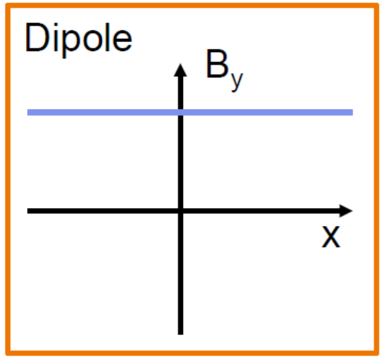
Storage Ring Magnets

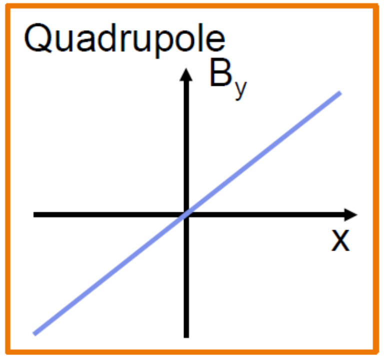
Bending magnets (dipoles)
have uniform constant vertical magnetic field $B_y = B_0$
They bend the beam and they define the circular trajectory.

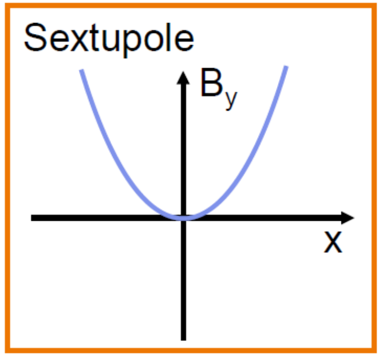
Quadrupoles:
magnetic field is linear with the distance from the center $B_y = K_1 x$
They are used to focus the beam.

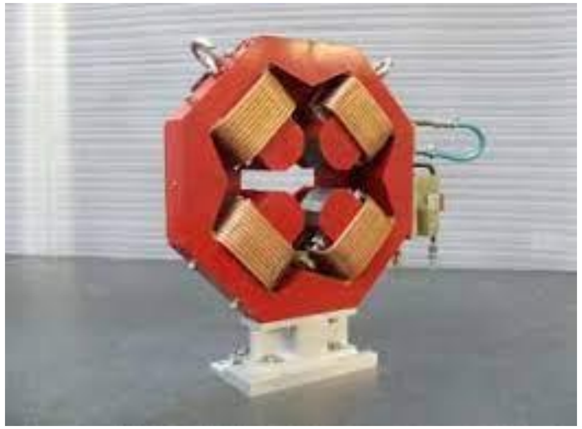
Sextupoles:
magnetic field is quadratic with the distance from the center $B_y = K_2 x^2$
They are used to correct chromatic effects.



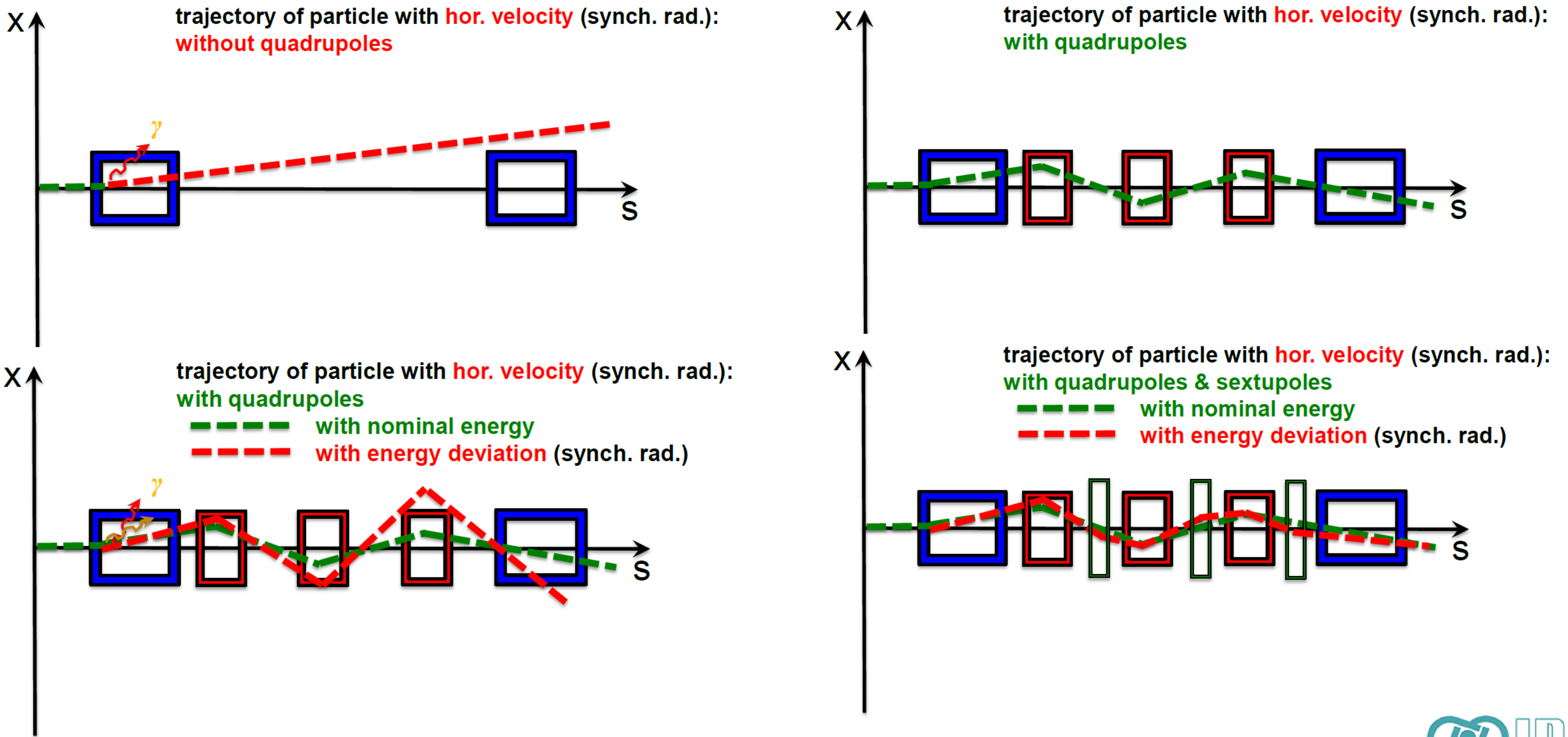






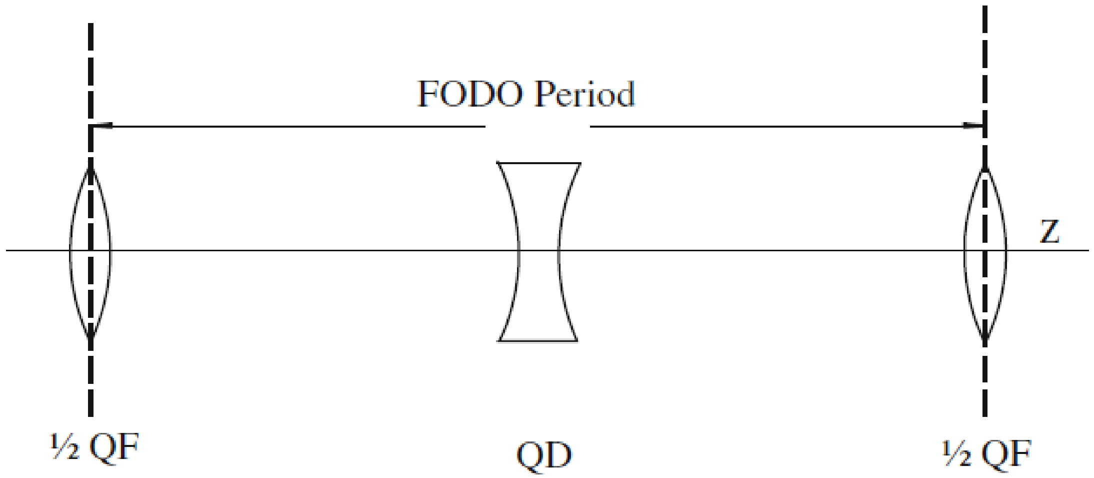


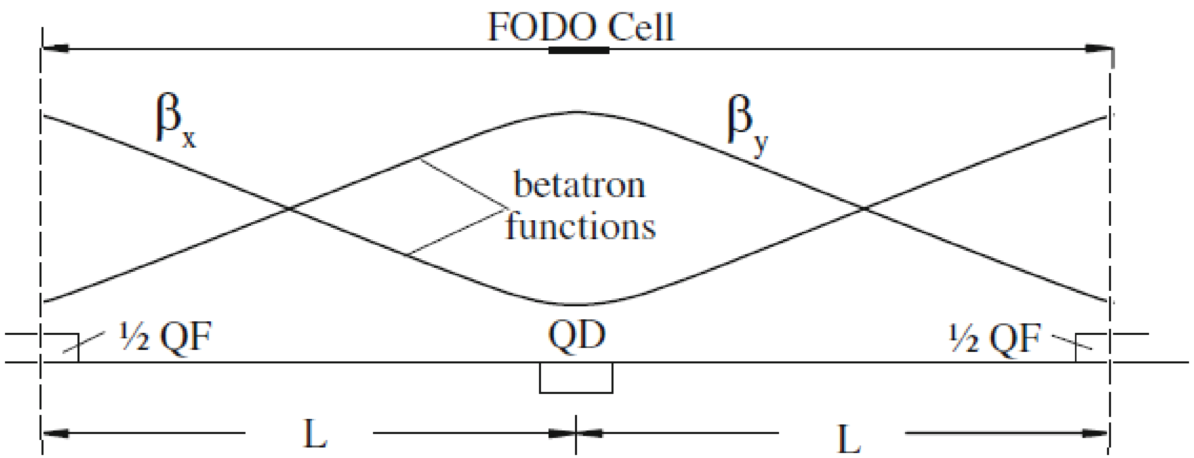
Storage Ring Magnets



FODO Lattice

The most simple periodic lattice would be a sequence of equidistant focusing quadrupoles of equal strength.
Each half of such a lattice period is composed of a focusing (F) and a defocusing (D) quadrupole with a drift space (O) in between forming a FODO sequence.
The FODO lattice is the most widely used lattice especially in high energy accelerator systems because of its simplicity, flexibility, and its beam dynamical stability.





FODO Lattice

OPA Lattice Design Code

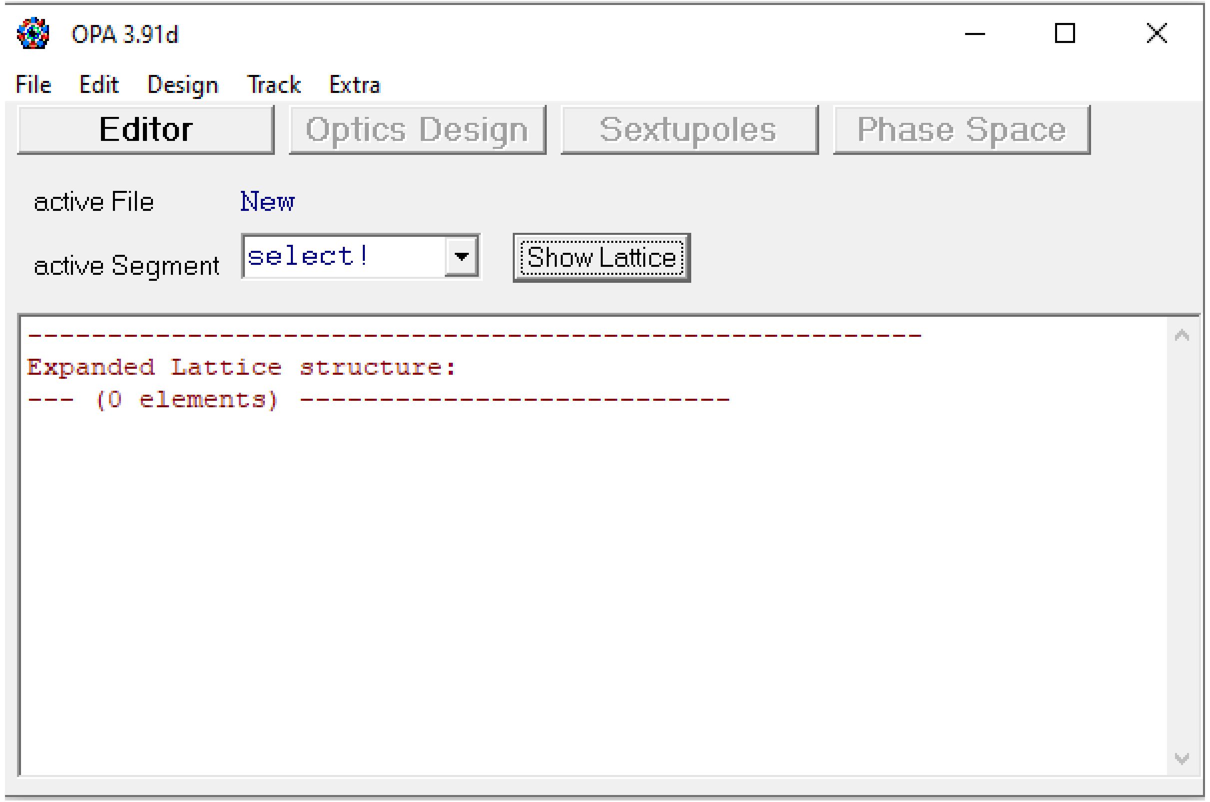
Linear Lattice Design

http://ados.web.psi.ch/opa/

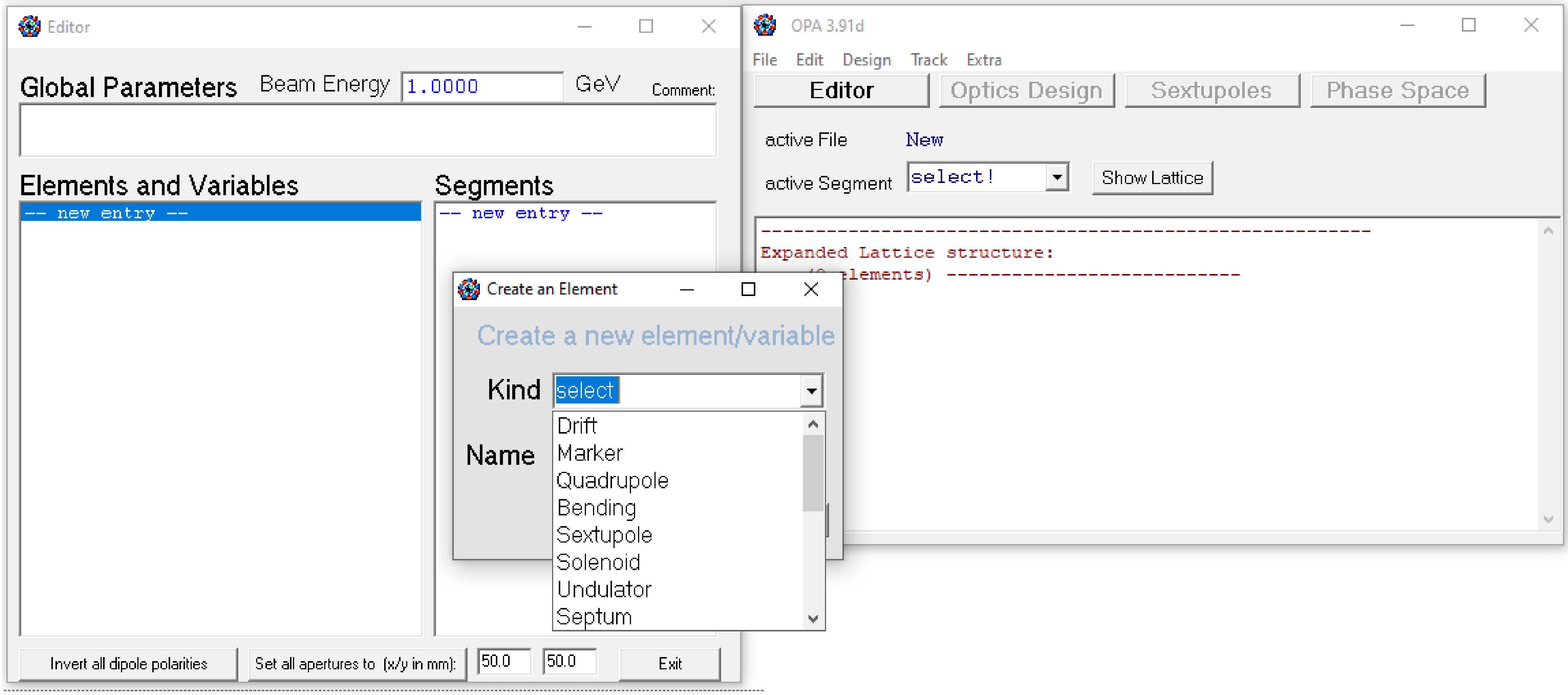
FODO parameter κ by

$$\kappa = \frac{f}{L} > 1 \qquad f^{-1} = k\ell\,.$$

Drift	1
BE length	1
QU length	0.2
K	1
Bending angle	10
N period	18



OPA Lattice Design



OPA Lattice Design

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Energy = 1.000000;

    BetaX   = 14.0725120; AlphaX  = 0.0000000;
    EtaX    = 6.6010999; EtaXP   = 0.0000000;
    BetaY   = 5.1734989; AlphaY  = 0.0000000;
    EtaY    = 0.0000000; EtaYP   = 0.0000000;

{----- Variables ---------------------------------------------------}


{----- Table of elements
---------------------------------------------------}

DR : Drift, L = 1.000000, Ax = 50.00, Ay = 50.00;

QF : Quadrupole, L = 0.100000, K = 1.000000, Ax = 50.00, Ay = 50.00;
QD : Quadrupole, L = 0.200000, K = -1.000000, Ax = 50.00, Ay = 50.00;

BE : Bending, L = 1.000000, T = 10.000000, K = 0.000000, T1 = 5.000000,
     T2 = 5.000000, Ax = 50.00, Ay = 50.00;


{----- Table of segments
---------------------------------------------------}

CELL : QF, DR, BE, DR, QD, DR, BE, DR, QF, NPER=18;

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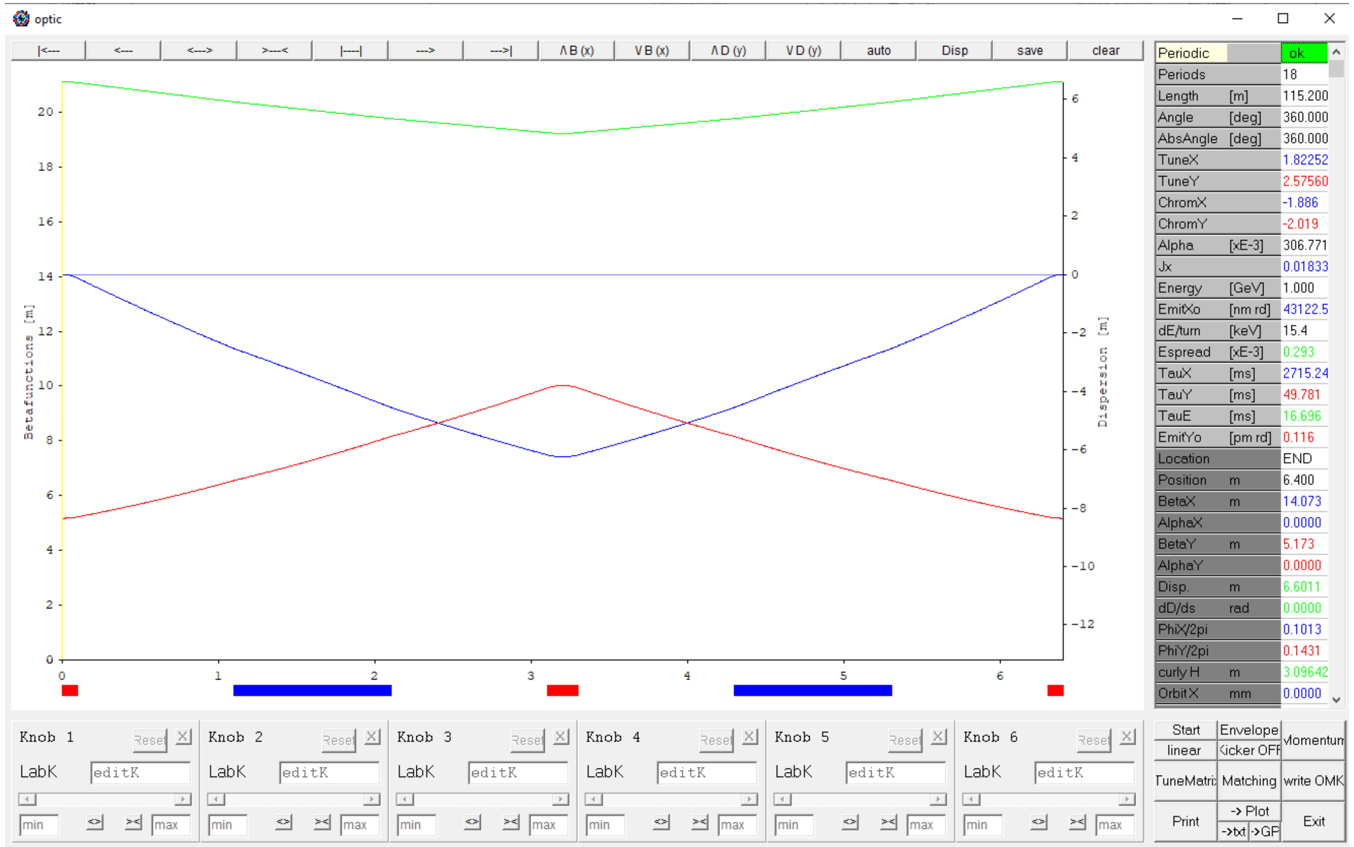

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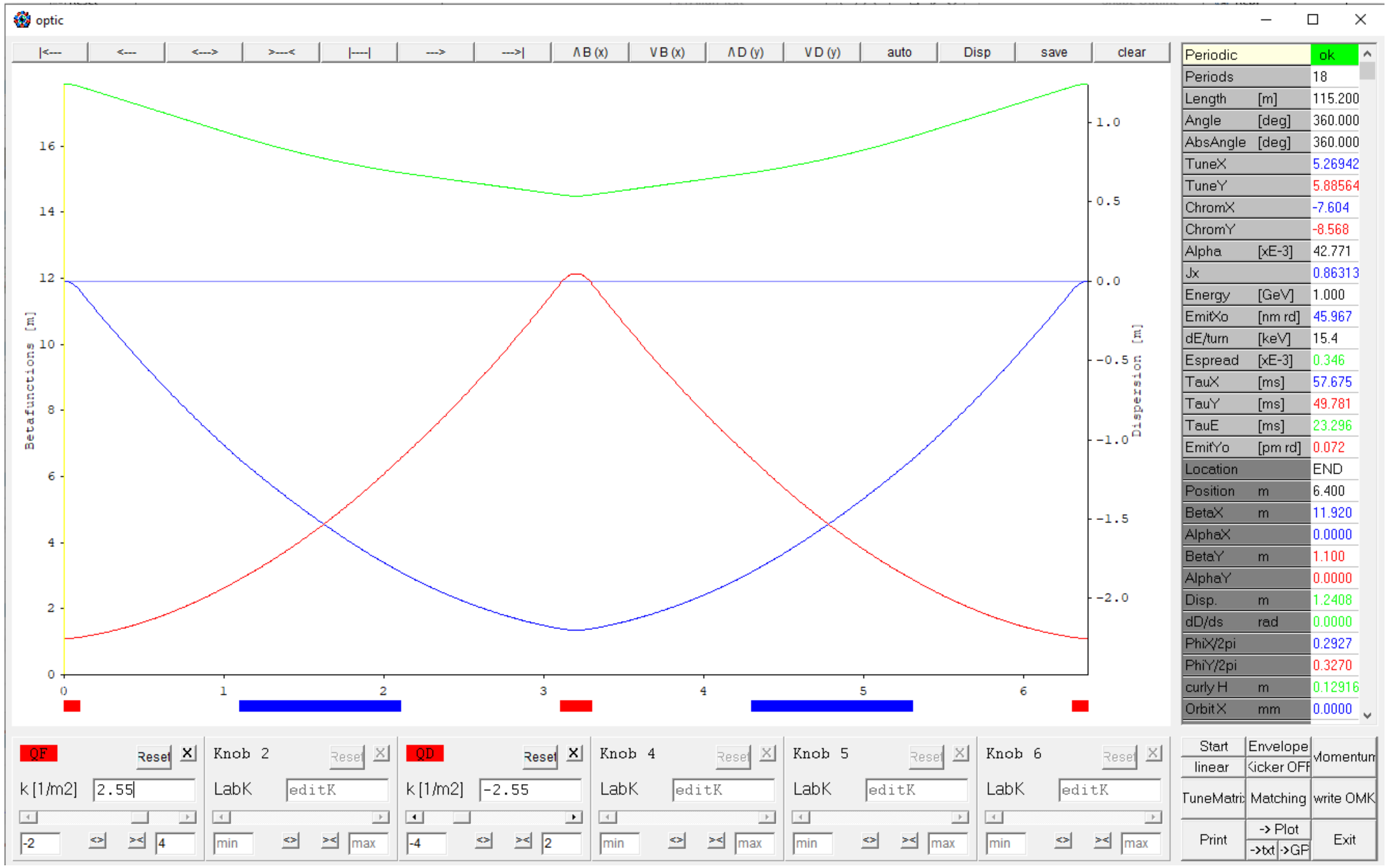
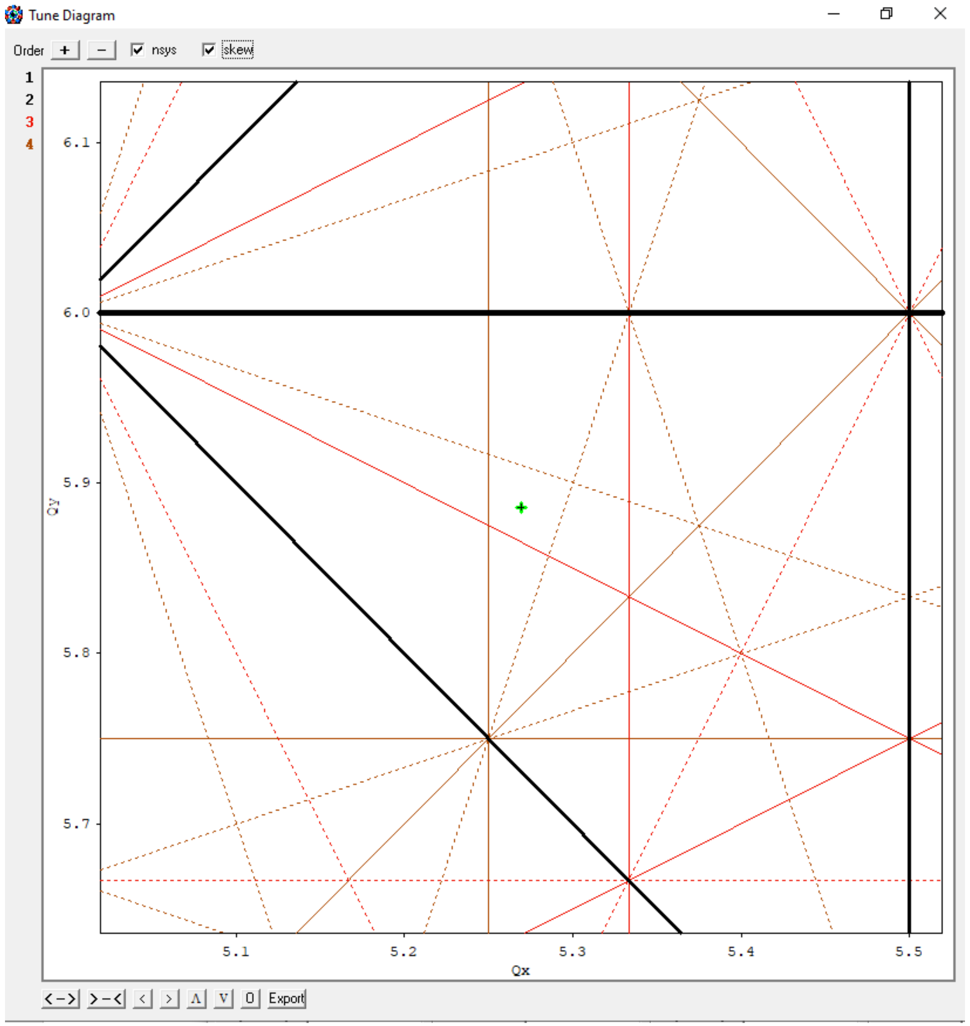
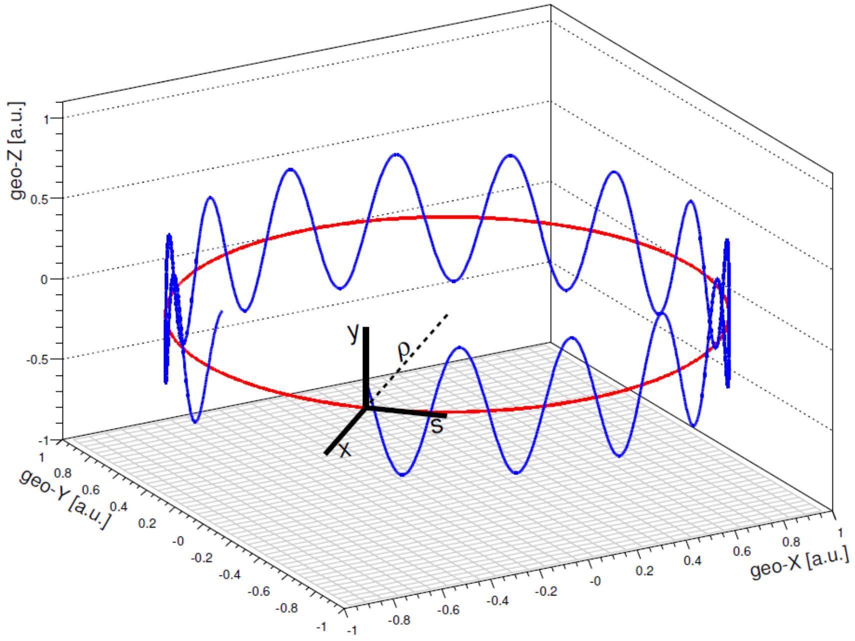
OPA Lattice Design





Tune Diagram

Betatron Oscillation

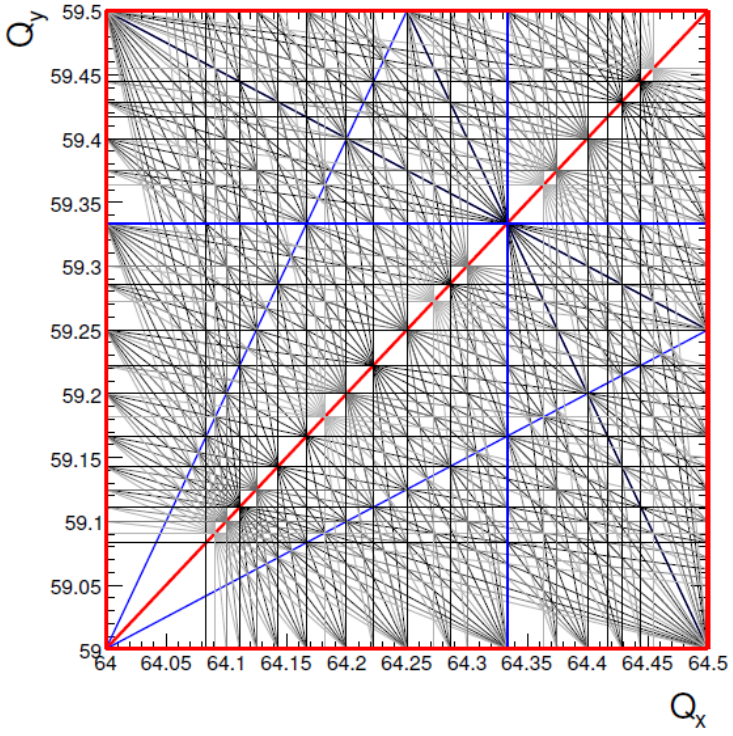


The number of transverse oscillations a particle describes per turn as **tune Q** of the machine.

$$Q := \frac{1}{2\pi} \oint_C \mu(s) \mathrm{d}s \qquad Q = Q_{\mathrm{int}} + q_{\mathrm{frac}}$$

resonance condition : $m \cdot Q_x + n \cdot Q_y = p$

The order of the resonance is given by |m| + |n|.

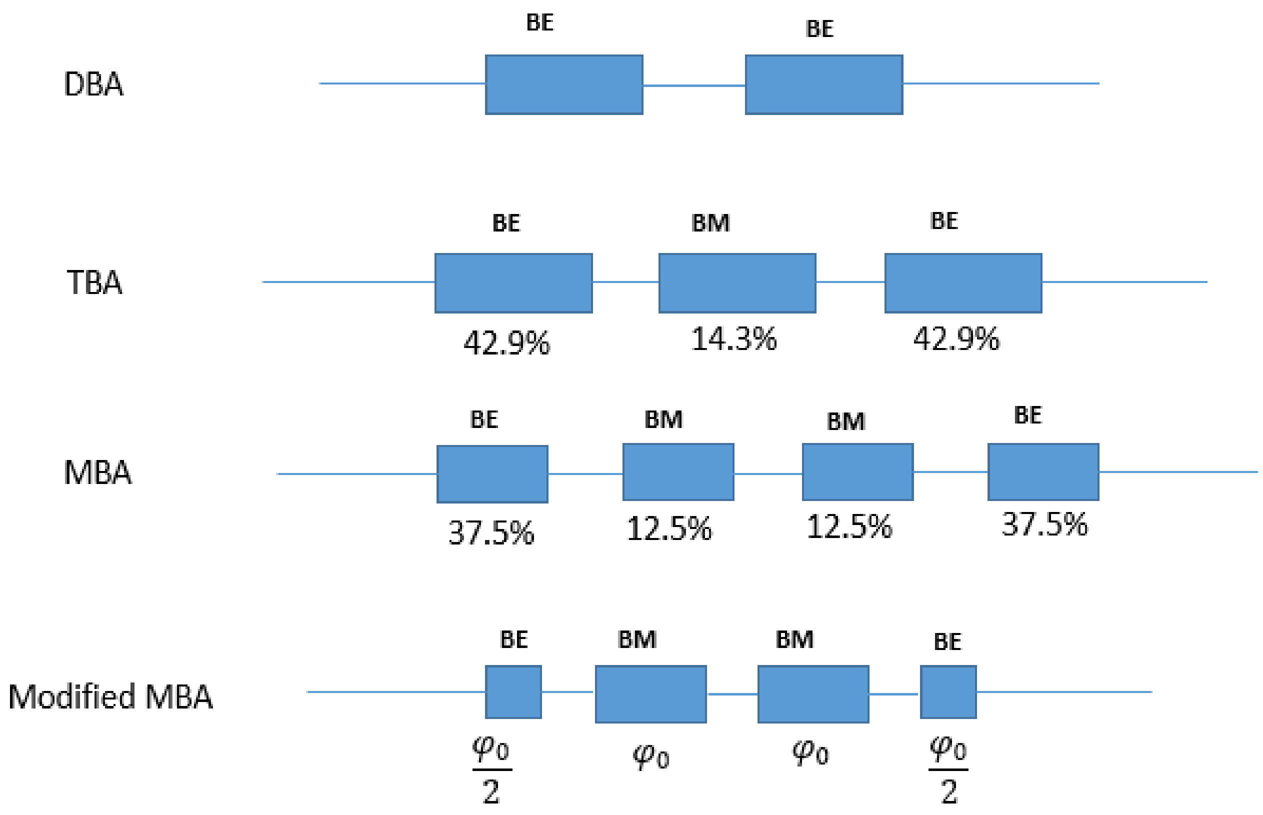


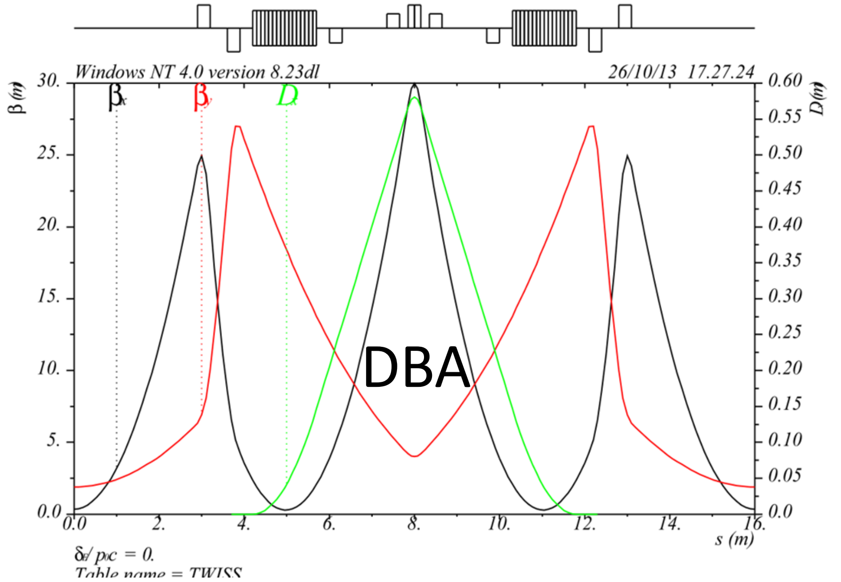
Lattice Structure

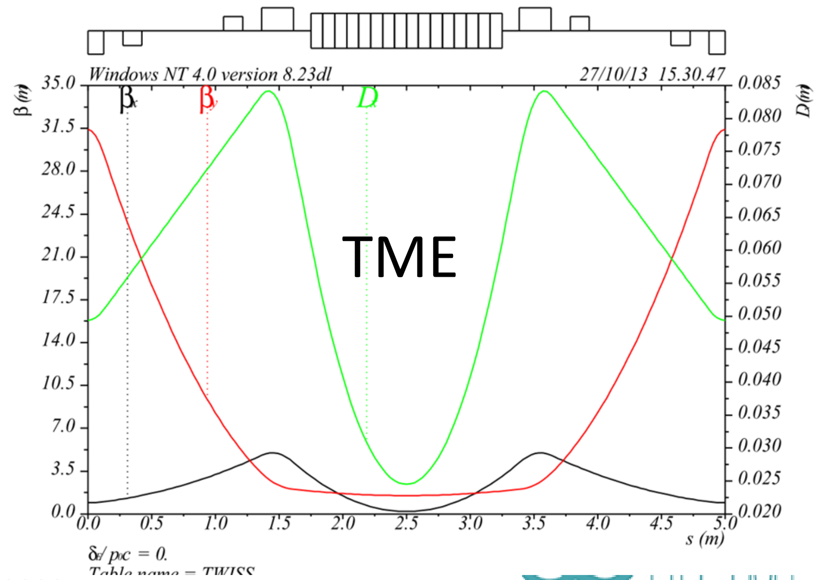
$\epsilon = F C_q \gamma^2 \theta^3$ $\qquad C_q \approx 3.832 \times 10^{-13}$

Lattice Style	F	Conditions
90^o FODO	$2\sqrt{2}$	$f = L/\sqrt{2}$
137^o FODO	1.2	Minimum emittance FODO
DBA	$\frac{1}{4\sqrt{15}}$	$\beta_{x,0} \approx \sqrt{12/5}L$, $\alpha_{x,0} \approx \sqrt{15}$
TME	$\frac{1}{12\sqrt{15}}$	$\eta_{x,min} \approx \frac{L\theta}{24}, \beta_{x,min} \approx L/2\sqrt{15}$
MBA	$\frac{1}{12\sqrt{15}}\left(\frac{M+1}{M-1}\right)$	M dipoles (with same radius of curvature) per cell

Lattice Structure

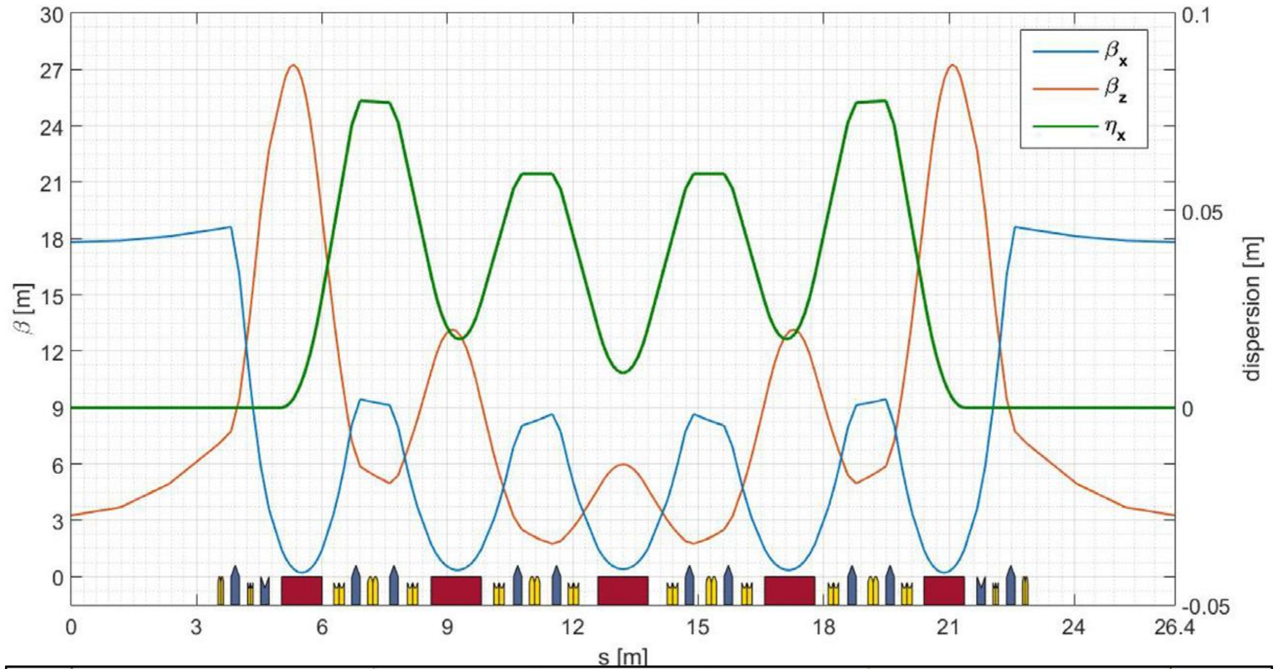






ILSF – MBA Lattice

- MBA lattice proposed for the first time in 1998.
- MBA structure satisfies at the same time all the criteria for pushing down the emittance.
- MBA needs much stronger magnets to squeeze beam dimension to much smaller values It means that we need small aperture vacuum pipe.



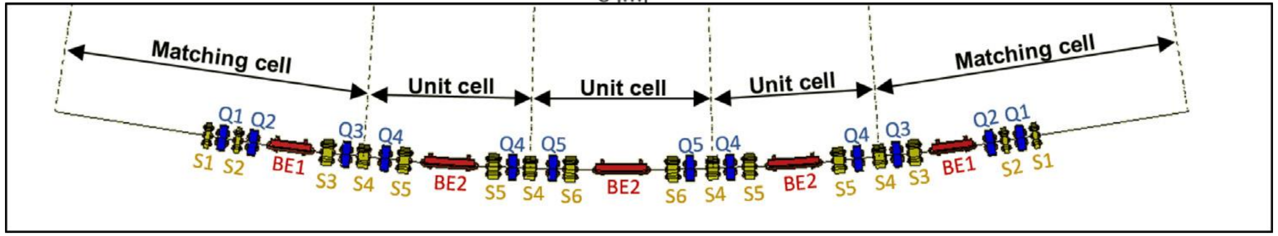
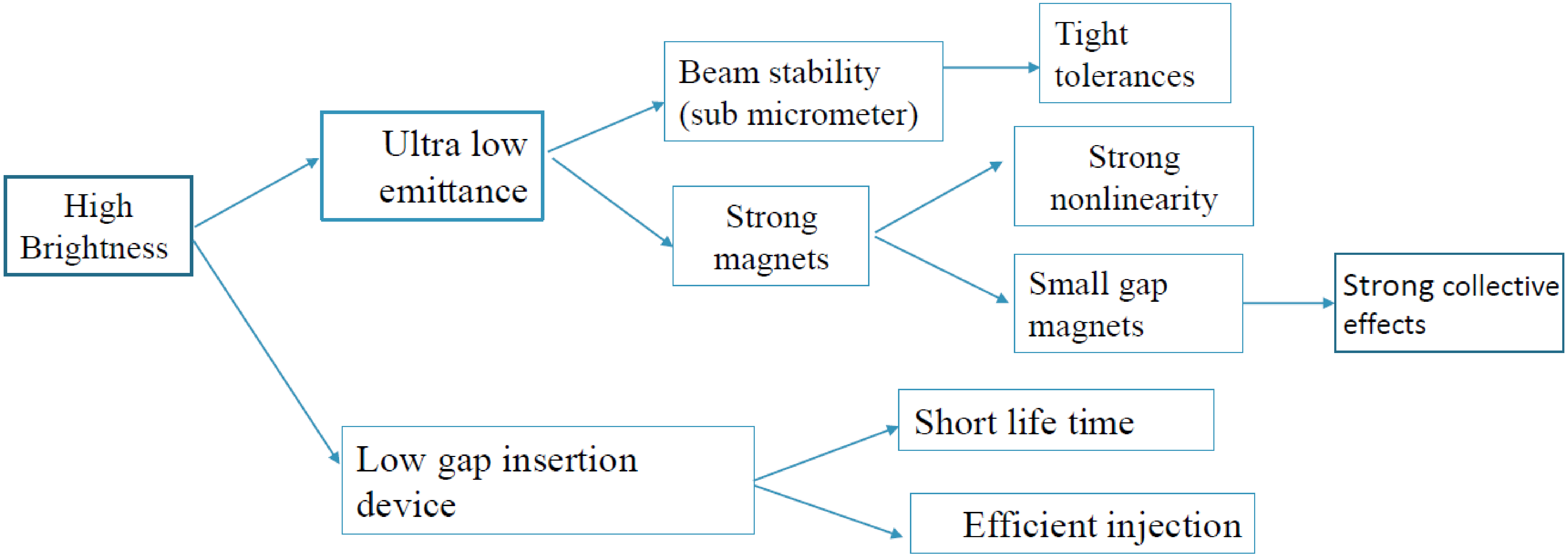


Table 1
Main parameters of ILSF storage ring.

Parameter	Symbol	Unit	Value
Energy	E	GeV	3
Circumference	C	m	528
Number of super period	–	–	20
Length of straight section		m	7.021
Natural emittance	ε	pm rad	270
Betatron tune	Q_x/Q_y		44.16/16.20
Natural chromaticity	ξ_x/ξ_y		−107.79/−61.30
1st order momentum compaction factor	α_c		1.824×10^{-4}
Natural energy loss per turn	U_0	keV	406.4
Natural energy spread	Δ		6.79×10^{-4}
Damping times	$\tau_x/\tau_y/\tau_s$	ms	18.857/26.002/16.039
Radiation integral, I_1	I_1	m	9.631×10^{-2}
Radiation integral, I_2	I_2	1/m	3.564×10^{-1}
Radiation integral, I_3	I_3	1/m^2	2.021×10^{-2}
Radiation integral, I_4	I_4	1/m	-1.350×10^{-1}
Radiation integral, I_5	I_5	1/m	1.003×10^{-5}
beta function at straight section	β_x/β_y	m/m	17.787/3.294
Min/Max horizontal beta function	$\beta_{x\_max}/\beta_{x\_min}$	m/m	0.207/18.608
Min/Max vertical beta function	$\beta_{y\_max}/\beta_{y\_min}$	m/m	1.740/27.195
Min/Max horizontal dispersion	η_{xMin}/η_{xMax}	cm/cm	0.000/7.776
RF frequency		MHz	100

Challenges of Multi Bend Achromat lattice



Challenges of Multi Bend Achromat lattice

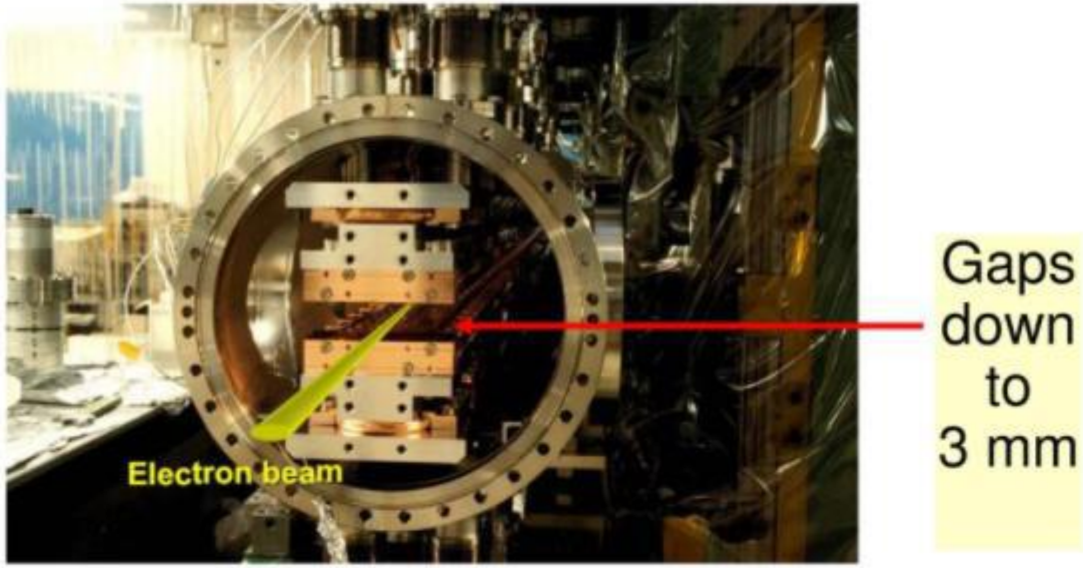
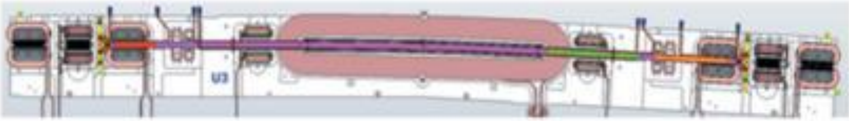
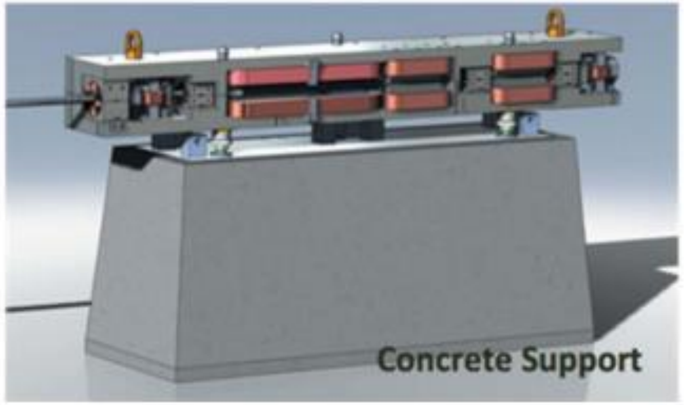


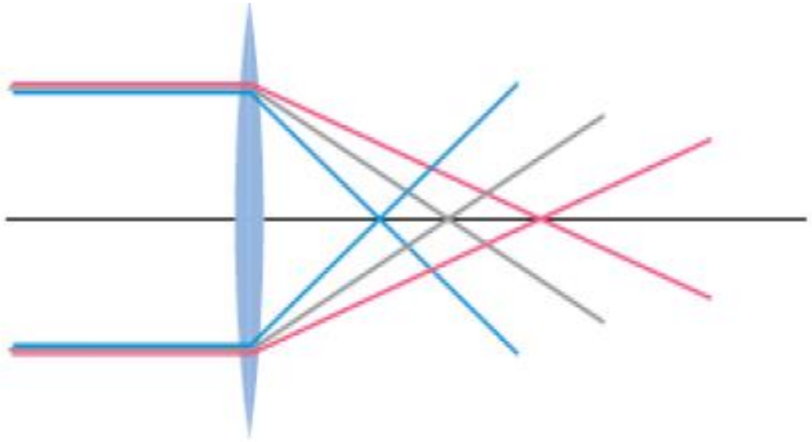
Fig. 8 Vacuum chamber comparison between 3GSR and 4GSR in DIAMOND

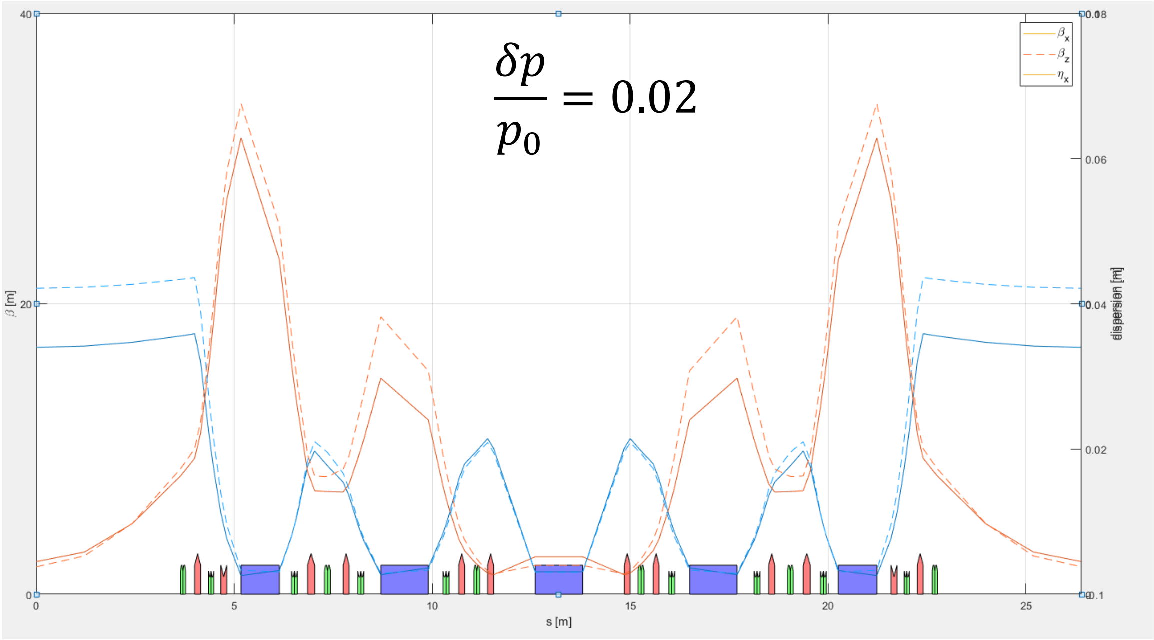




Nonlinear optimization

Quadrupole focusing strength depends on the particle momentum





Natural chromaticity:

$$\xi^{(1)}_{x.y} = \frac{\Delta\nu}{\frac{\Delta p}{p}} < 0$$

For ILSF case: $\xi_x = -115.38$

$\xi_y = -74.66$

For $\frac{\Delta p}{p}$=1% → $\Delta\nu_x = 1.15$

$\Delta\nu_x = 0.75$

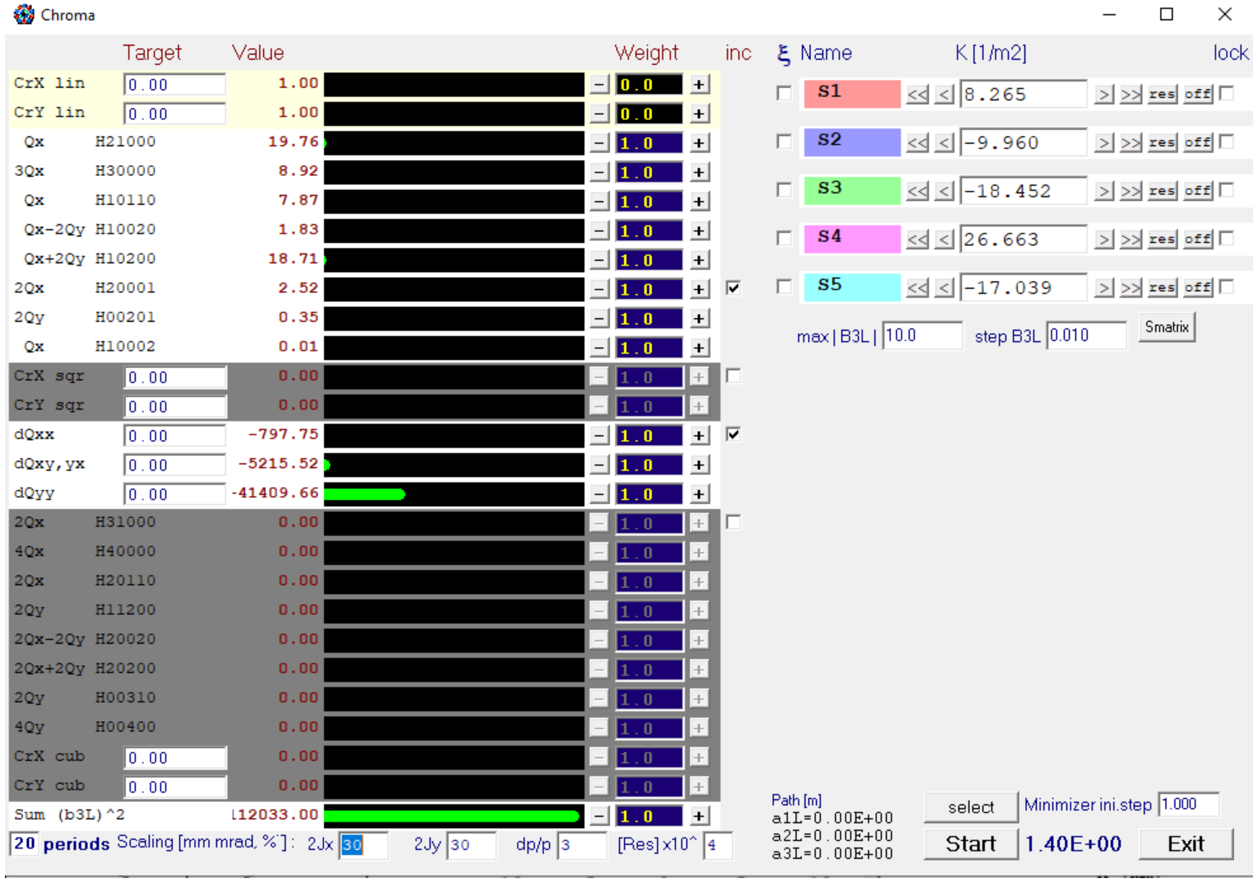
Chromaticity is and index shows how much strong quadrupoles have been used in the lattice

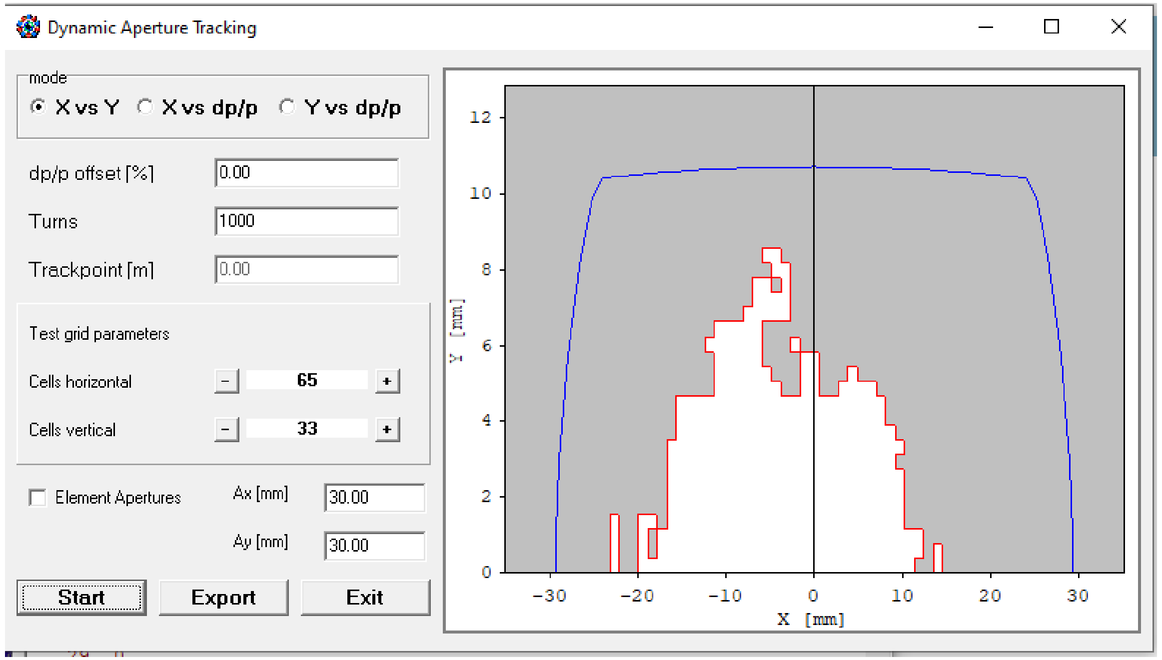
Nonlinear optimization

Using the sextupole fields will make the dynamics of electrons nonlinear

Sextupole field : $(B_x = gxy, B_y = \frac{1}{2}g(x^2 - y^2))$

Produce geometric aberration, higher order chromaticity and resonance deriving terms





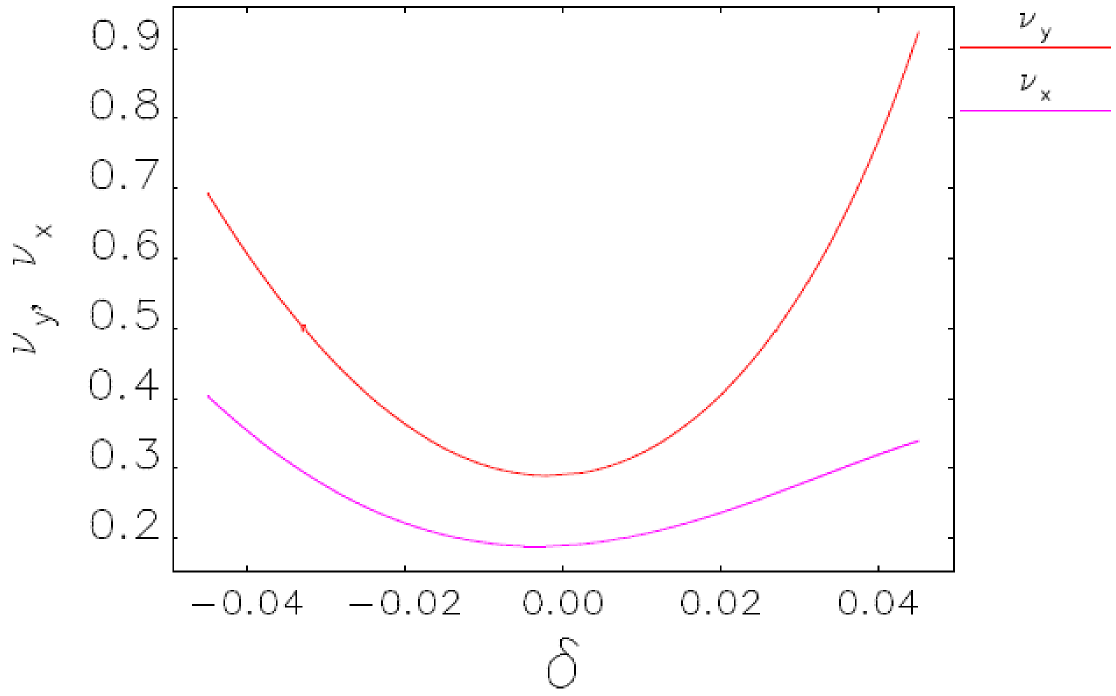
Nonlinear optimization

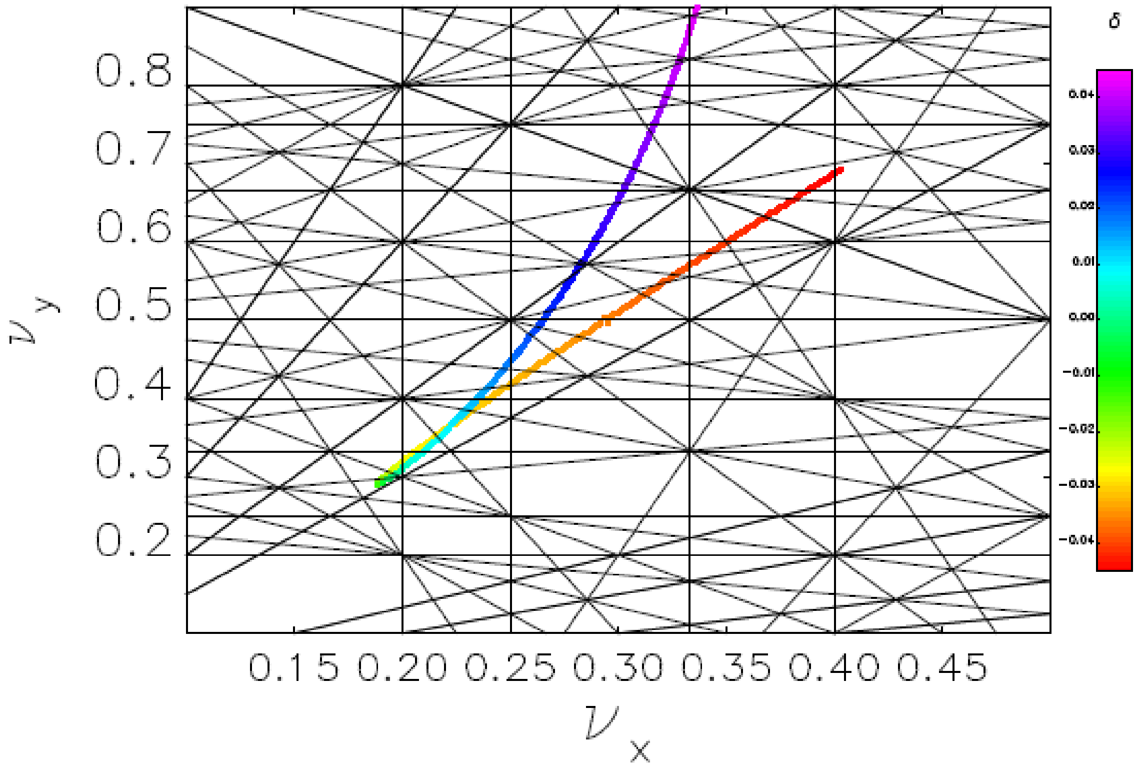
- Traditional approach to nonlinear dynamics optimization:
  - varying the strengths and positions of different sextupole families
  - adding higher-order multipole magnets
  - changing the fractional betatron tunes
  - varying the lattice functions at locations of nonlinear magnets

the optimization must be performed with the aid of a computer program such as OPA, elegant, AT, MAD-X,...

Momentum and Amplitude Detuning

- to improve the nonlinear behavior of the lattice, one needs to minimize tune shifts with amplitude and momentum, as well as the strength of driving terms for nearby resonances





Momentum and Amplitude Detuning

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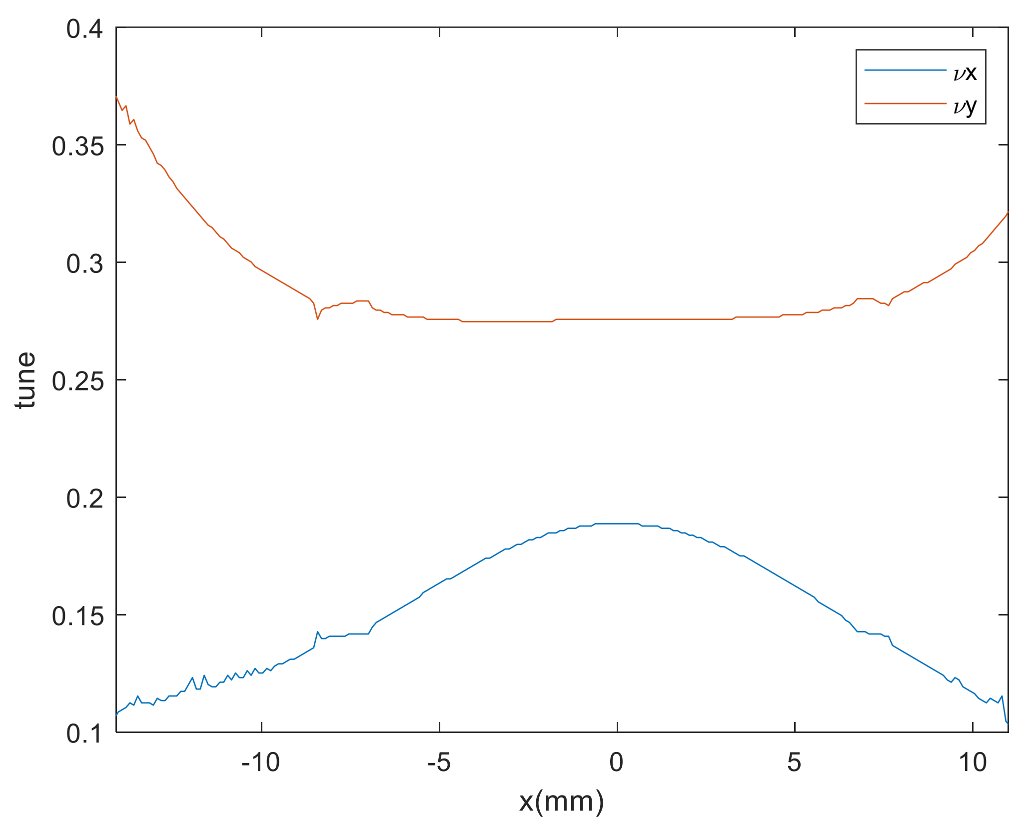

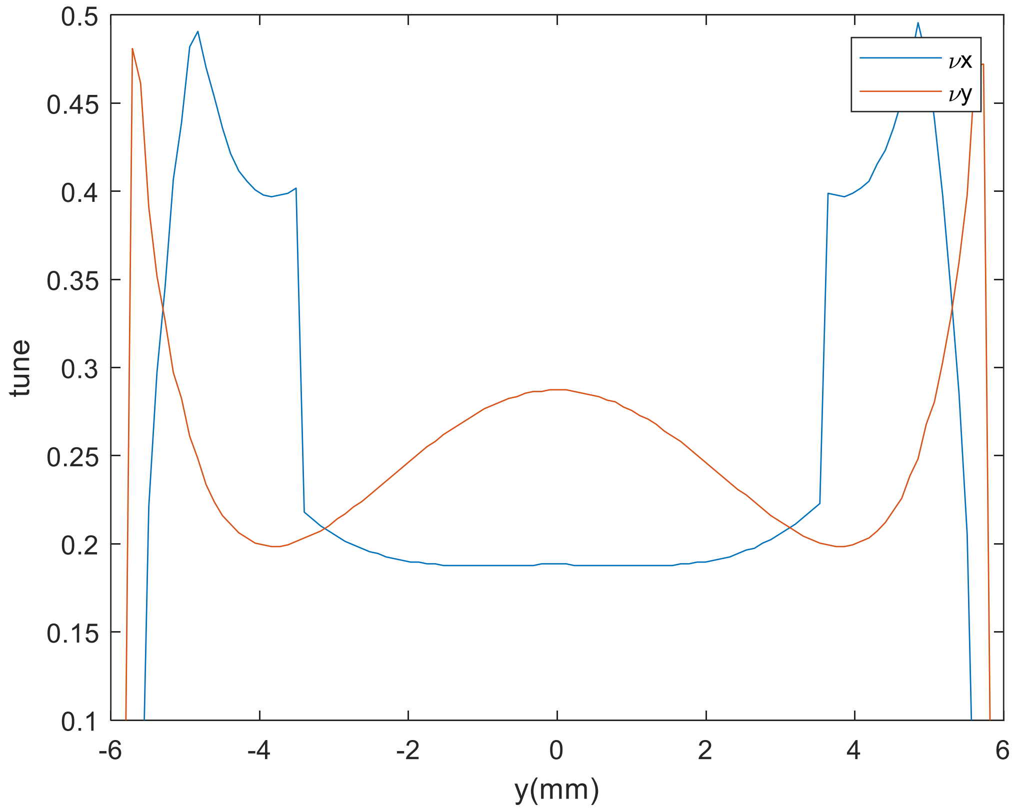

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Thank you for your attention