Supersymmetric continuous spin gauge theory

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Tehran

Based on:

M. Najafizadeh, " Supersymmetric Continuous Spin Gauge Theory "

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Outline:

- Supersymmetry (SUSY)
- Continuous Spin Particle (CSP)
- Continuous spin gauge field theory
- SUSY CSP

convention: $p^2 = 0\,, \quad \mathcal{N} = 1\,, \quad d = 4$

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- a symmetry between fermions and bosons
- SUSY particles have not yet been observed
- zero-point energy
- supercharge Q: $\quad Q\,|\,\text{boson}> \,=\, |\,\text{fermion}> \qquad Q\,|\,\text{fermion}> \,=\, |\,\text{boson}>$
- supermultiplet: $\quad (\text{ boson }, \text{ fermion })$

  where $\quad \#\text{ bosonic d.o.f} = \#\text{ fermionic d.o.f}$

$$(0\,,\,\tfrac{1}{2}) \longrightarrow \text{chiral multiplet (Wess-Zumino, 1974)}$$
$$(1\,,\,\tfrac{1}{2}) \longrightarrow \text{gauge multiplet}$$
$$(2\,,\,\tfrac{3}{2}) \longrightarrow \text{gravity multiplet}$$
$$\vdots$$
$$(\ s\ ,\ s+\tfrac{1}{2}\ ) \longrightarrow \text{half-integer spin multiplet (Curtright, 1979)}$$
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$$S_{_{SUSY}} = S_b\,[\phi] + S_f\,[\psi]$$

one should find "SUSY transformations" : $\begin{cases} \delta\phi &= \bar{\epsilon}\,(\ \cdots\ )\,\psi \\ \delta\psi &= \ (\ \cdots\ )\,\phi\,\epsilon \end{cases}$ such that

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Example: chiral multiplet (0 , 1/2):

Naively, consider action principles of a **real scalar** and **Majorana spinor** fields

$$S \;=\; S_b + S_f \;=\; \tfrac{1}{2}\int d^4x\;\left[\,(\partial_\mu\,\phi)(\partial^\mu\,\phi)\;+\;\bar{\psi}\,(\,i\partial\!\!\!/\,)\,\psi\,\right]$$

One can show that the above action is invariant under transformations

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where $\approx$ denotes the equation of motion $(i\partial\!\!\!/\psi = 0)$ is used (on-shell supersymmetry)

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By defining the complex scalar field

$$\Phi = \frac{1}{\sqrt{2}}\,(\phi \,-\, i\,\tilde{\phi}) \qquad \Phi^\dagger = \frac{1}{\sqrt{2}}\,(\phi \,+\, i\,\tilde{\phi})$$

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2. is a unitary irreducible representation (UIRs) of the Poincaré group
3. characterizes by "continuous spin parameter" μ (with the dimension of a mass)
4. representation decomposes into a direct sum of all helicity reps., at $\mu = 0$
5. has infinite degrees of freedom per space-time point
6. wave equations were introduced by Wigner
7. has two types of representation; bosonic & fermionic

   Bosonic CSP field:

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   Fermionic CSP field:

   $$\Psi(x,\eta) = \Psi(x) + \eta^\mu\,\Psi_\mu(x) + \tfrac{1}{2!}\,\eta^\mu\eta^\nu\,\Psi_{\mu\nu}(x) + \tfrac{1}{3!}\,\eta^\mu\eta^\nu\eta^\rho\,\Psi_{\mu\nu\rho}(x)\cdots$$

8. states are NOT boost invariant (unlike the helicity states) while single-CSP is

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Historically, constructing a covariant action principle for CSP has been a mystery for decades, however, about 75 years after Wigner's classification

- The first bosonic CSP action (Schuster & Toro, 2014) unconstrained ✓
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- ⋮
- SUSY CSP (on-shell)

2017	Zinoviev	in frame-like	(N=1 , d=3)
2019	Buchbinder et al.	in frame-like	(N=1 , d=4)
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Supersymmetric continuous spin gauge theory

(SUSY CSP)

SUSY CSP

- In all SUSY theories, (# bosonic d.o.f) = (# fermionic d.o.f)
- However, a bosonic or fermionic CSP has infinite number of d.o.f
- hence, in a CSP supermultiplet there would be four possibilities:

$$\mathcal{N}=1 \text{ CSP supermultiplet } \Rightarrow \Big(\quad \text{CSP} \quad , \quad \text{CSPino} \quad \Big)$$

$$\Downarrow$$

$$\begin{array}{lll} (\text{ real CSP} , & \text{Majorana CSP}) & \times \\ (\text{complex CSP} , & \text{Majorana CSP}) & \times \\ (\text{ real CSP} , & \text{Dirac CSP}) & \times \\ (\text{complex CSP} , & \text{Dirac CSP}) & \checkmark \end{array}$$

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SUSY CSP: unconstrained formalism

Unconstrained formalism of the SUSY CSP action is given by

$$S^{SUSY}_{CSP} = S^{b}_{CSP} + S^{f}_{CSP} = \int d^4x\, d^4\eta \Big[\Phi^\dagger(x,\eta)\ \mathbf{B}\ \Phi(x,\eta)\ +\ \overline{\Psi}(x,\eta)\ \mathbf{F}\ \Psi(x,\eta)\Big]$$

where
$$\begin{cases} \mathbf{B} = \delta'(\eta^2+1)\Big[-\Box + (\eta\cdot\partial_x)(\partial_\eta\cdot\partial_x+\mu) - \frac{1}{2}(\eta^2+1)(\partial_\eta\cdot\partial_x+\mu)^2\Big] \\ \mathbf{F} = \delta'(\eta^2+1)\Big[(\not{\eta}\ + i)\,\not{\partial} - (\eta^2+1)(\partial_\eta\cdot\partial_x+\mu)\Big] \end{cases}$$

SUSY trans.
$$\begin{cases} \delta\,\Phi(x,\eta) = \frac{1}{\sqrt{2}}\,\bar\epsilon\,(1+\gamma^5)\,(\not{\eta}\ - i\,)\,\Psi(x,\eta) \\ \delta\,\Psi(x,\eta) = \frac{1}{\sqrt{2}}\,\big[\not{\partial} - \frac{1}{2}\,(\not{\eta}\ + i\,)(\partial_\eta\cdot\partial_x+\mu\,)\big]\,(1-\gamma^5)\,\epsilon\,\Phi(x,\eta) \end{cases}$$

SUSY algebra
$$\begin{cases} [\,\delta_1\,,\,\delta_2\,]\,\Phi(x,\eta) = -2i\,(\,\bar\epsilon_2\,\not{\partial}\,\epsilon_1\,)\,\Phi(x,\eta) & \checkmark \\ [\,\delta_1\,,\,\delta_2\,]\,\Psi(x,\eta) \approx -2i\,(\,\bar\epsilon_2\,\not{\partial}\,\epsilon_1\,)\,\Psi(x,\eta)\ +\ \text{gauge trans.} & \checkmark \end{cases}$$

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$$\text{where} \quad \begin{cases} \mathbf{B} \; = -\,\Box \; + \; 5 \text{ terms} \; + \; \mu\,(6 \text{ terms}) \; + \; \mu^2\,(4 \text{ terms}) \\ \mathbf{F} \; = \; i\,\partial\!\!\!/ \; + \; 6 \text{ terms} \; + \; \mu\,(7 \text{ terms}) \end{cases}$$

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At $\mu = 0$, CSP supermultiplet decomposes into

$$\left(0\,,\; \tfrac{1}{2} \right) \;\oplus\; \sum_{s=1}^{\infty} \left(s\,,\; s+\tfrac{1}{2} \right) \;\oplus\; \sum_{s=0}^{\infty} \left(s+\tfrac{1}{2}\,,\; s+1 \right)$$

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$$S_{CSP}^{SUSY} = S_{CSP}^{b} + S_{CSP}^{f} = \int d^4x \left[\Phi^\dagger \ \mathbf{B} \ \Phi \ + \ \overline{\Psi} \ \mathbf{F} \ \Psi \right]$$

$$\text{where} \begin{cases} \mathbf{B} = -\Box \ + \ 5 \text{ terms} \ + \ \mu \, (6 \text{ terms}) \ + \ \mu^2 \, (4 \text{ terms}) \\ \mathbf{F} = \ i \, \partial\!\!\!/ \ + \ 6 \text{ terms} \ + \ \mu \, (7 \text{ terms}) \end{cases}$$

$$\text{SUSY trans.} \begin{cases} \delta\Phi = \bar{\epsilon} \left(\frac{1+\gamma^5}{\sqrt{2}} \ + \ 1 \text{ term} \right) \Psi \\ \delta\Psi = \left(-\frac{i}{\sqrt{2}} \, \partial\!\!\!/ (1-\gamma^5) \ + \ 6 \text{ terms} \ + \ \mu \, (5 \text{ term}) \right) \epsilon \, \Phi \end{cases}$$

At $\mu = 0$, CSP supermultiplet decomposes into

$$\left(0 \, , \, \tfrac{1}{2} \right) \ \oplus \ \sum_{s=1}^{\infty} \left(s \, , \, s + \tfrac{1}{2} \right) \ \oplus \ \sum_{s=0}^{\infty} \left(s + \tfrac{1}{2} \, , \, s + 1 \right)$$

Current work & Open problems

Current work at IPM

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Open problems

- group-theoretical meaning of CSP in AdS
- investigation of CSP in condensed matter systems
- AdS/CFT correspondence for CSP
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- ...

Thank you for your attention!

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