Observable Quantum Loops in the Sky

Aliakbar Abolhasani
Sharif university of Technology
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In Collaboration with **M.M.Sheikh-Jabbari**

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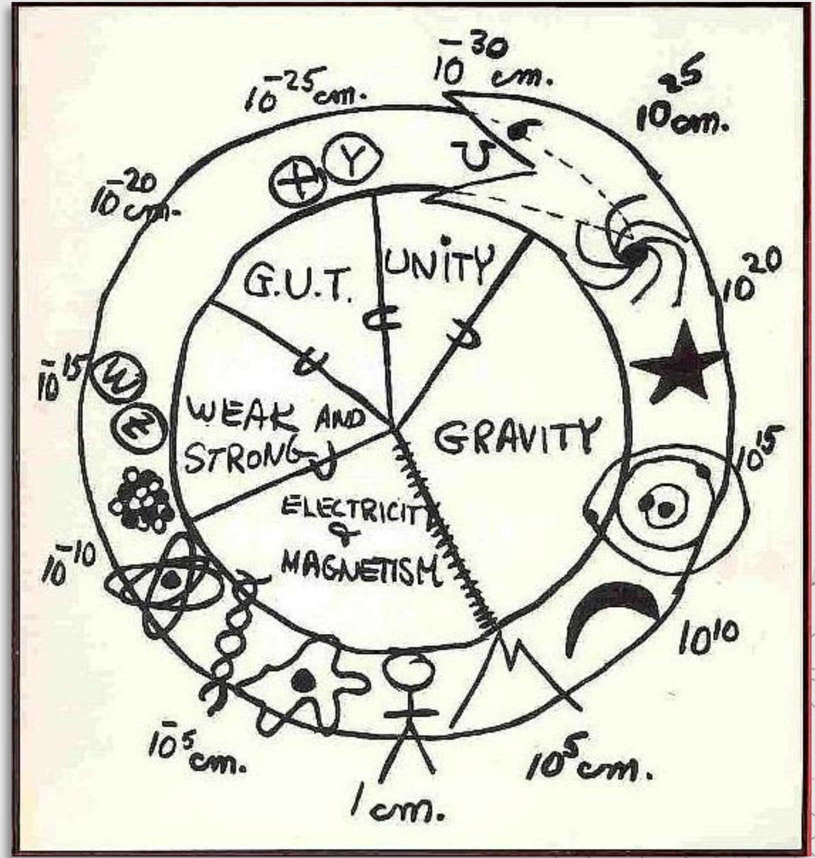
Background

Current status of Cosmology

Early Universe Cosmology



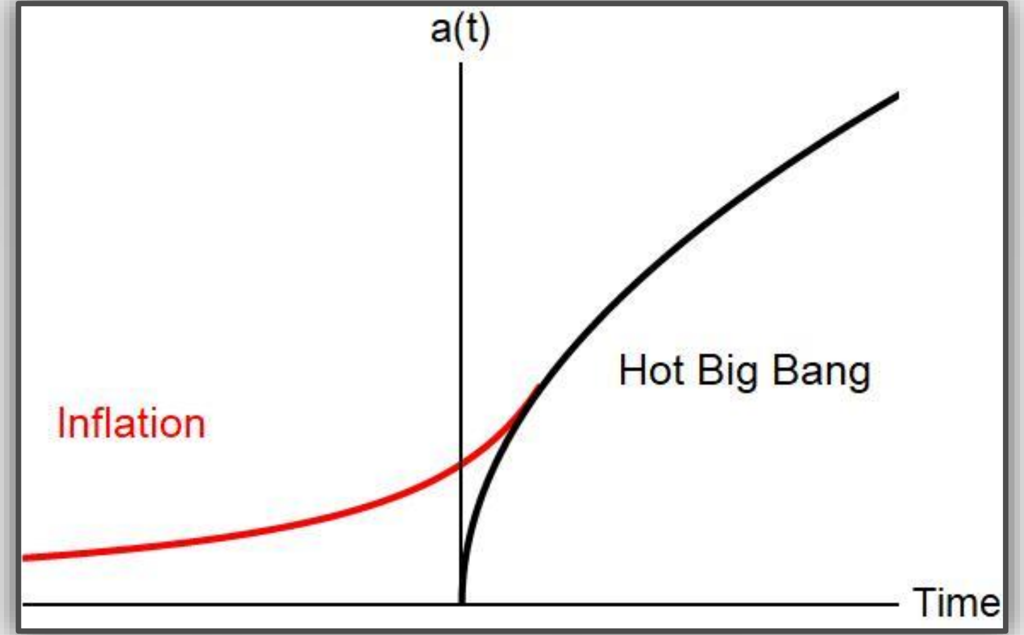
The very early universe becomes, in Zel'dovich's words, "an accelerator for poor people" that can give us some rough information about fundamental physics.

What Came First: Inflation or BigBang?

Today it is widely believed that the universe underwent a period of (near) exponential expansion at very onset of its evolution

The Universe is filled with a homogeneous background of inflaton field.

Though widely accepted there are powerful rivals.



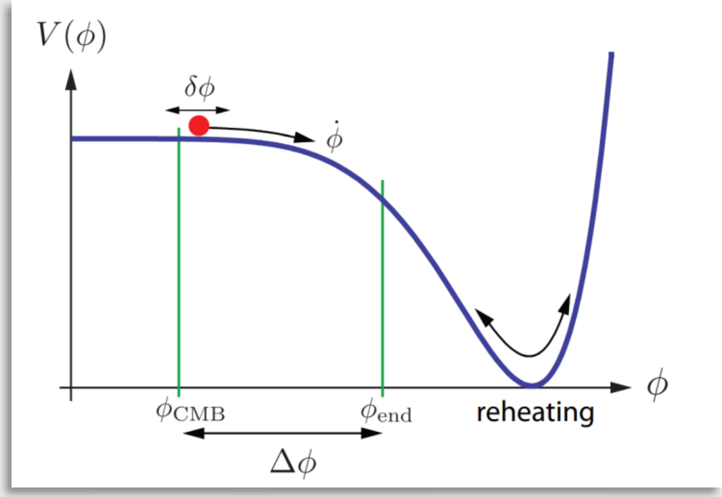
James Peebles won this year's Nobel prize in physics for helping transform the field of cosmology into a respected science, but if there's one term he hates to hear, it's "Big Bang Theory.“ (Phys.org)

A Simple Model

In the simplest scenarios of the cosmic inflation, the universe at the very onset of its evolution is assumed to be filled with a homogeneous background of scalar field called inflaton.

If this scalar field rolls slowly towards the minimum of its potential the universe expands nearly exponentially.

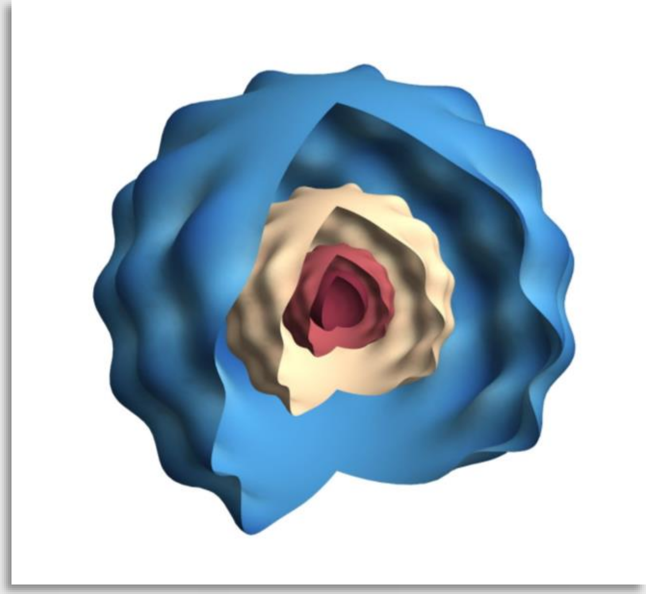
Credit: Daniel Baumann



Inhomogeneities

Scalar perturbations freeze once their wave-length exceeds the horizon size, while fluctuations of massive scalar fields decay.

These perturbations –by causing an inhomogeneous expansion-- make ripples on the fiber of the space-time.



In-In vs In-out

Calculating cosmological correlation functions is different from usual Q.F.T calculations in various respects (S.W. 2005):

- We are interested in evaluating quantum expectation values rather than scattering amplitudes.
- Boundary conditions are not imposed on +/- infinity but they are fixed at a very early times where --by equivalence principle-- they are the same as those of a Minkowski space.
- Due to time dependence of background parameter the Hamiltonian that governs the evolution of fluctuations has an explicit time dependence.

In-in vs Iteration

The main project is to calculate quantum expectation value of a product of some fields

$$\langle \hat{O}(t) \rangle = \left\langle \left[\bar{T} \exp \; i \int_{-\inf}^{t} H_I(t)\, dt \right] O(t) \left[T \exp -i \int_{-\inf}^{t} H_I(t)\, dt \right] \right\rangle$$

by either

- Conventional in-in formalism: Expanding right and left exponentials
- Iterative solution of quantum operators

$$\hat{O}_N(\tau) = i^N \int \prod_{i=1}^{N} \frac{d\tau_i}{H^4 \tau_i^4} \theta(\tau_{i-1} - \tau_i) \int \prod_{i=1}^{N} d^3\boldsymbol{y}_i \left[H_{int}(\tau_N, \boldsymbol{y}_N), ..., \left[H_I(\tau_2, \boldsymbol{y}_2), \left[H_I(\tau_1, \boldsymbol{y}_1), \hat{O}_0(\tau) \right] \right] \right]$$

02

Amplification Module

Extra degrees of freedom with a mass modulated with inflaton

Extrad Field with Modulated Mass

Inflaton potential has a discrete shift symmetry $\phi \to \phi + 2\pi f$.

Additional (massive) field coupled to the inflaton with masses modulated by the periodic term in

$$m_\chi^2 = \mu^2 + 2g^2 f^2 \cos \phi / f$$

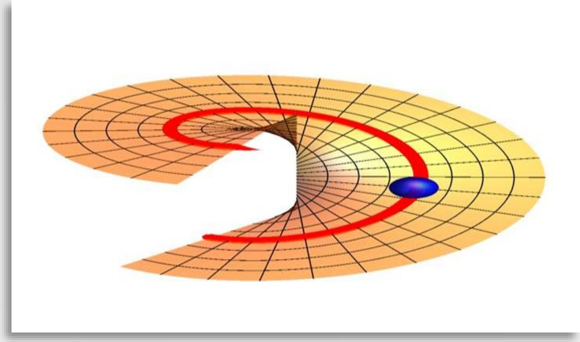
This would introduce a new energy scale into the problem

$$\boxed{\alpha \equiv \frac{\omega}{H} = \frac{1}{H}\frac{\dot{\phi}}{f}}$$

The state of universe is

$$|\Psi\rangle = \exp\left(-\frac{1}{2}\int d^3\mathbf{p}\beta_n(p)a^\dagger(\mathbf{p})a^\dagger(-\mathbf{p})\right)|0\rangle$$

Remark. We can safely ignore the back-reaction effects during inflation for an appropriate choice of parameters of the model.



Our Model

The full Lagrangian of the model is

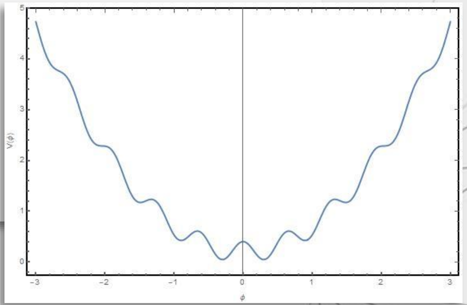
$$\mathcal{L} = -\frac{1}{2}(\partial\phi)^2 - \frac{1}{2}(\partial\chi)^2 - \Lambda^4\left((\frac{\phi}{f})^p + b\cos(\frac{\phi}{f})\right) - \frac{1}{2}m_\chi^2(\phi)\chi^2.$$

The action for scalar perturbations is

$$\mathcal{L} = -\frac{1}{2}(\partial\varphi)^2 - \frac{3}{2}\eta H^2\varphi^2 - \frac{1}{2}(\partial\chi)^2 - \frac{1}{2}\mathcal{G}f^2\cos(\omega t)\chi^2 + \mathcal{L}_{\mathrm{int.}} + \mathcal{O}(\varphi^3),$$

With

$$\mathcal{L}_{int.} \simeq \frac{1}{2}\mathcal{G}f\sin\omega t\varphi\chi^2 + \frac{1}{4}\mathcal{G}\cos\omega t\varphi^2\chi^2,$$



Remark. Only cubic interaction is relevant for our calculations here.

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Diagrammatic Approach

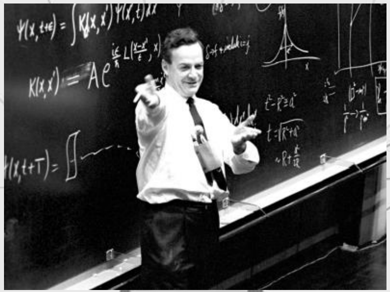
Feynman rules and Correlation Functions

Pros/Cons

- It ususally make the calculation much more lengthy compared to usual in-in calculations.

- It is a useful tool for systematic calculation of correlation functions particularly higher order terms of perturbation theory.

- It is a great help for estimating the superficial degree of divergence of correlation function at all order.



Feynman Rules

- Zeroth order fields

$= \varphi_0(\boldsymbol{y})$

$= \chi_0(y)$

- Propagators

$= G_\varphi(\boldsymbol{x}, \boldsymbol{y})$

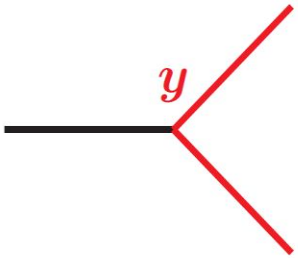
$= G_\chi(\boldsymbol{x}, \boldsymbol{y})$

where

$$G_\chi(\tau, \tau'; \boldsymbol{x}, \boldsymbol{x}') = i\theta(\tau - \tau')\left[\chi(\tau, \boldsymbol{x}), \chi(\tau', \boldsymbol{x}')\right].$$

Interaction Vertex

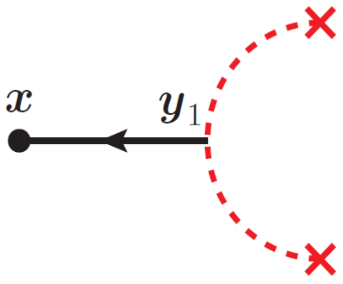
We have cubic interaction term



$$= -\int d^4\boldsymbol{y}\, g_3(\tau_y) = -\int d^3\boldsymbol{y}\; d\mathcal{T}_y$$

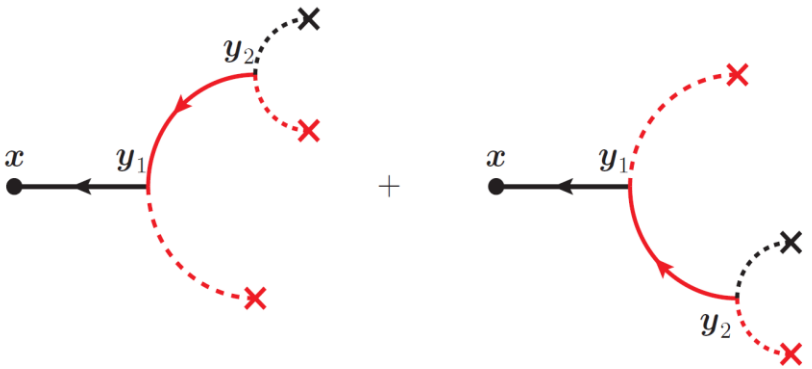
where $d\mathcal{T} \equiv \dfrac{d\tau}{H^4\tau^4}\, g_3(\tau).$

$\varphi_1(\tau, \boldsymbol{x}) =$



$$= -\int d^3\boldsymbol{y}_1\, d\mathcal{T}_1\ G_\varphi(\tau, \tau_1; \boldsymbol{x}, \boldsymbol{y}_1)\chi^2(\tau_1, \boldsymbol{y}_1)$$

$\varphi_2(\tau, \boldsymbol{x}) =$



$$= 2\int d^3\boldsymbol{y}_1 d^3\boldsymbol{y}_2\, d\mathcal{T}_1 d\mathcal{T}_2\ \varphi(\tau_2, \boldsymbol{y}_2)G_\varphi(\tau, \tau_1; \boldsymbol{x}, \boldsymbol{y}_1)G_\chi(\tau_1, \tau_2; \boldsymbol{y}_1, \boldsymbol{y}_2)\{\chi(\tau_1, \boldsymbol{y}_1), \chi(\tau_2, \boldsymbol{y}_2)\}$$

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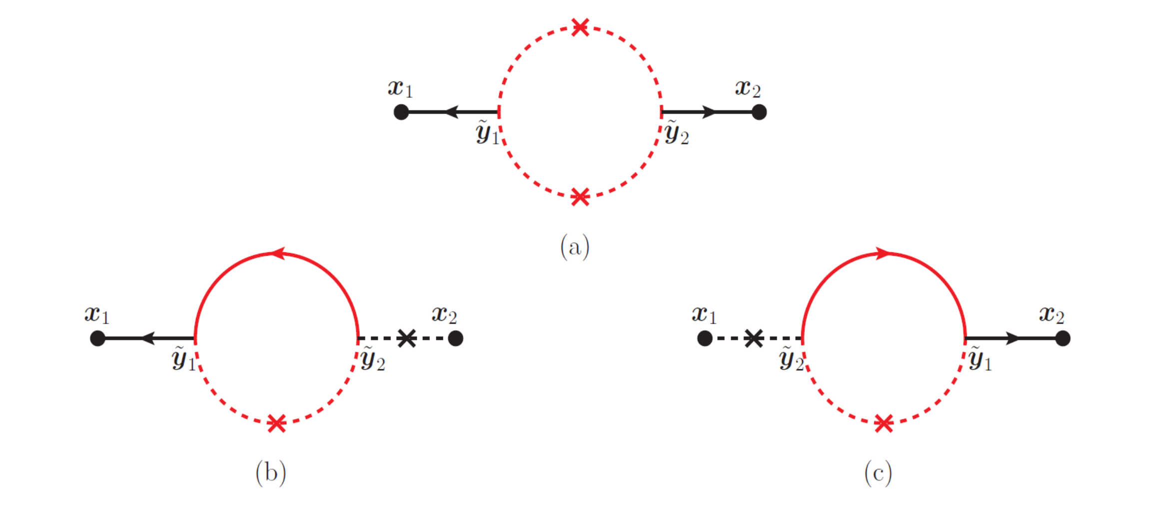
Correlation Functions

Power spectrum and Bispectrum of the scalar perturbations

Two point function: 1 -loop



$$P_{11}(\tau, x_1; \tau, x_2) \equiv \langle \varphi_1(\tau, x_1)\varphi_1(\tau, x_2)\rangle =$$

$$2 \times \int d^3\tilde{y}_1 d^3\tilde{y}_2 \; d\tilde{\mathcal{T}}_1 d\tilde{\mathcal{T}}_2 G_\varphi(\tau, \tilde{\tau}_1; x_1, \tilde{y}_1) \; G_\varphi(\tau, \tilde{\tau}_2; x_2, \tilde{y}_2)\langle \hat{\chi}(\tilde{\tau}_1, \tilde{y}_1)\hat{\chi}(\tilde{\tau}_2, \tilde{y}_2)\rangle^2$$

Time integrals and Momentum Cut-off

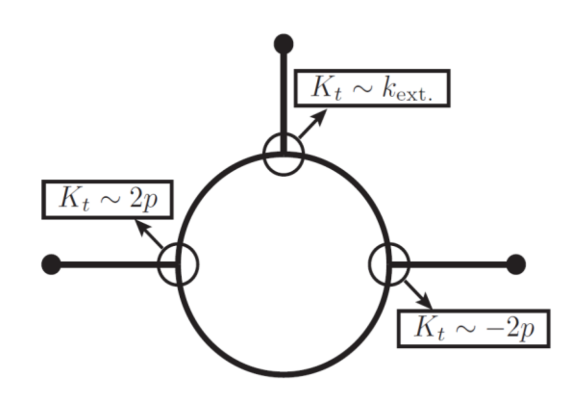
As analogy, the oscillating coupling can be though as a incoming/outgoing particle with physical four momentum $p^\mu = (\omega, 0, 0, 0)$.

$$\int^{\tau} \frac{d\tau'}{\tau'} \sin\omega t' \, e^{\pm i|K_t|\tau'} \propto e^{\pm i\Phi(|K_t|,\alpha)}$$

For $p \gg k$, the resonance happens at $\tau_s \simeq \dfrac{\alpha}{2p}$.

Due to causality, scalar fluctuations does not evolve on super horizon scales, namely for $k\tau \to 0$ propagators vanishes. This naturally cut the loop momentum at

$$\boxed{p < \alpha k}$$



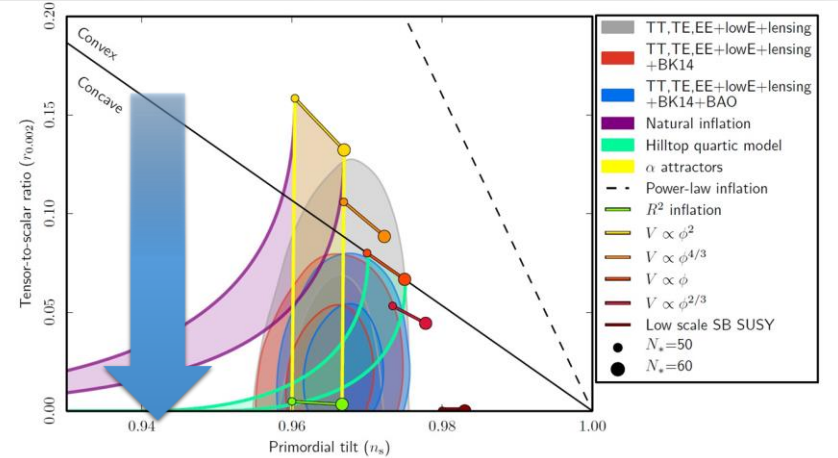
Reconciling Convex Models with Planck

power spectrum of curvature perturbations of this model is dominated by the one-loop contribution

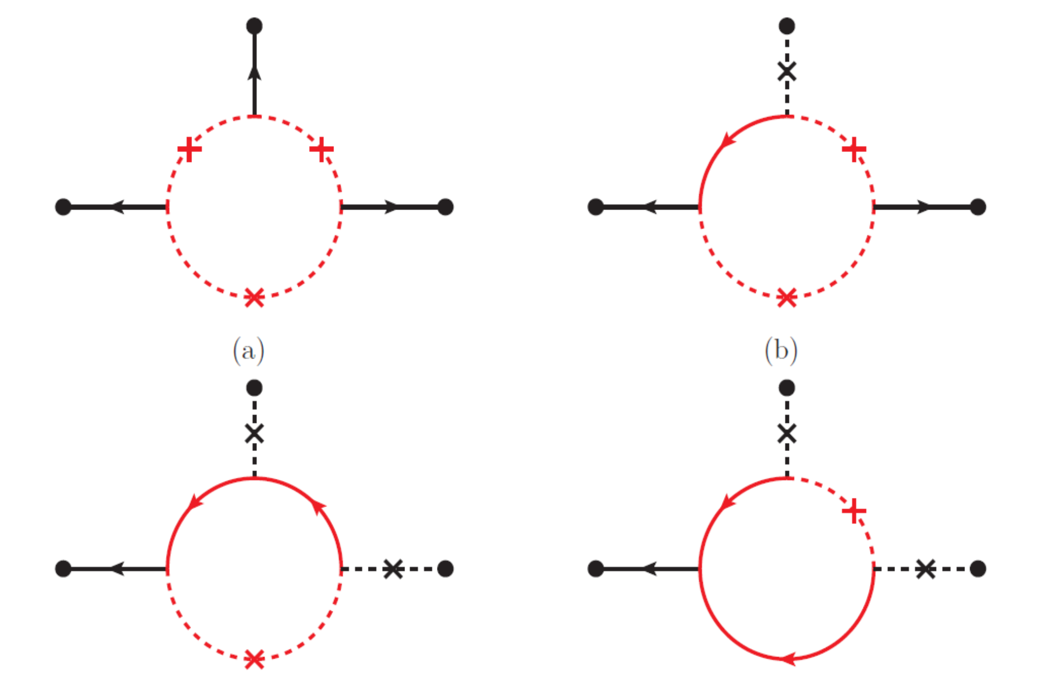
$$\frac{\langle \varphi_{\boldsymbol{k}} \varphi_{-\boldsymbol{k}} \rangle}{P_\phi(k)} \simeq \frac{\pi^3}{24} \left(\frac{\mathcal{G}^2 f^2}{H^2} \right) = \frac{\pi^3}{24} \left(\frac{\mathbb{H}}{H} \right)^2 .$$

This allows to disentangle the scale of scalar and tensor perturbations and hence to suppress the ratio of tensor-to-scalar power spectra

Besides, it alters the expression of scalar spectral tilt from the simple chaotic models, thus opening the way to reconcile chaotic models with convex potential and the Planck data.



Three point function at 1-loop



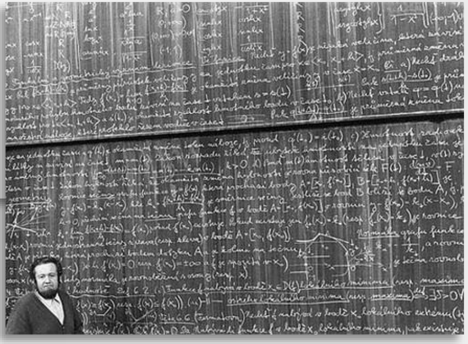
$$\begin{aligned}\langle\varphi(\boldsymbol{x}_1)\varphi(\boldsymbol{x}_2)\varphi(\boldsymbol{x}_3)\rangle =&\langle\varphi_1(\boldsymbol{x}_1)\varphi_1(\boldsymbol{x}_2)\varphi_1(\boldsymbol{x}_3)\rangle\\ &+\langle\varphi_2(\boldsymbol{x}_1)\varphi_1(\boldsymbol{x}_2)\varphi_0(\boldsymbol{x}_3)\rangle+\langle\varphi_2(\boldsymbol{x}_1)\varphi_0(\boldsymbol{x}_2)\varphi_1(\boldsymbol{x}_3)\rangle+\langle\varphi_1(\boldsymbol{x}_1)\varphi_2(\boldsymbol{x}_2)\varphi_0(\boldsymbol{x}_3)\rangle\\ &+\langle\varphi_1(\boldsymbol{x}_1)\varphi_0(\boldsymbol{x}_2)\varphi_2(\boldsymbol{x}_3)\rangle+\langle\varphi_0(\boldsymbol{x}_1)\varphi_2(\boldsymbol{x}_2)\varphi_1(\boldsymbol{x}_3)\rangle+\langle\varphi_0(\boldsymbol{x}_1)\varphi_1(\boldsymbol{x}_2)\varphi_2(\boldsymbol{x}_3)\rangle\\ &+\langle\varphi_3(\boldsymbol{x}_1)\varphi_0(\boldsymbol{x}_2)\varphi_0(\boldsymbol{x}_3)\rangle+\langle\varphi_0(\boldsymbol{x}_1)\varphi_3(\boldsymbol{x}_2)\varphi_0(\boldsymbol{x}_3)\rangle+\langle\varphi_0(\boldsymbol{x}_1)\varphi_0(\boldsymbol{x}_2)\varphi_3(\boldsymbol{x}_3)\rangle\end{aligned}$$

For example the amplitude B111 is

$$B_{111}(k_1,k_2,k_3) \equiv \langle \varphi_1(\boldsymbol{k}_1)\varphi_1(\boldsymbol{k}_2)\varphi_1(\boldsymbol{k}_3)\rangle$$
$$= 8P_\phi^{1/2}(k_1)P_\phi^{1/2}(k_2)P_\phi^{1/2}(k_3)\int d\mathcal{T}_1\, d\mathcal{T}_2\, d\mathcal{T}_3\ \mathrm{Re}\varphi_{\boldsymbol{k}_1}(\tau_1)\mathrm{Re}\varphi_{\boldsymbol{k}_2}(\tau_2)\mathrm{Re}\varphi_{\boldsymbol{k}_3}(\tau_3)\times$$
$$\left\langle(\hat{\chi}^2)_{\boldsymbol{k}_1}(\tau_1)(\hat{\chi}^2)_{\boldsymbol{k}_2}(\tau_2)(\hat{\chi}^2)_{\boldsymbol{k}_3}(\tau_3)\right\rangle$$

After a tedious but straightforward we get

$$B_\varphi(\boldsymbol{k}_1,\boldsymbol{k}_2,\boldsymbol{k}_3) = \frac{2\pi^{5/2}}{2\sqrt{2}}\frac{\mathcal{G}^3 f^3}{H^3}\frac{1}{k_1^2\, k_2^2\, k_3^2}\frac{\ln\alpha}{\alpha^{5/2}}\sin\Phi(2k_2,\alpha).$$



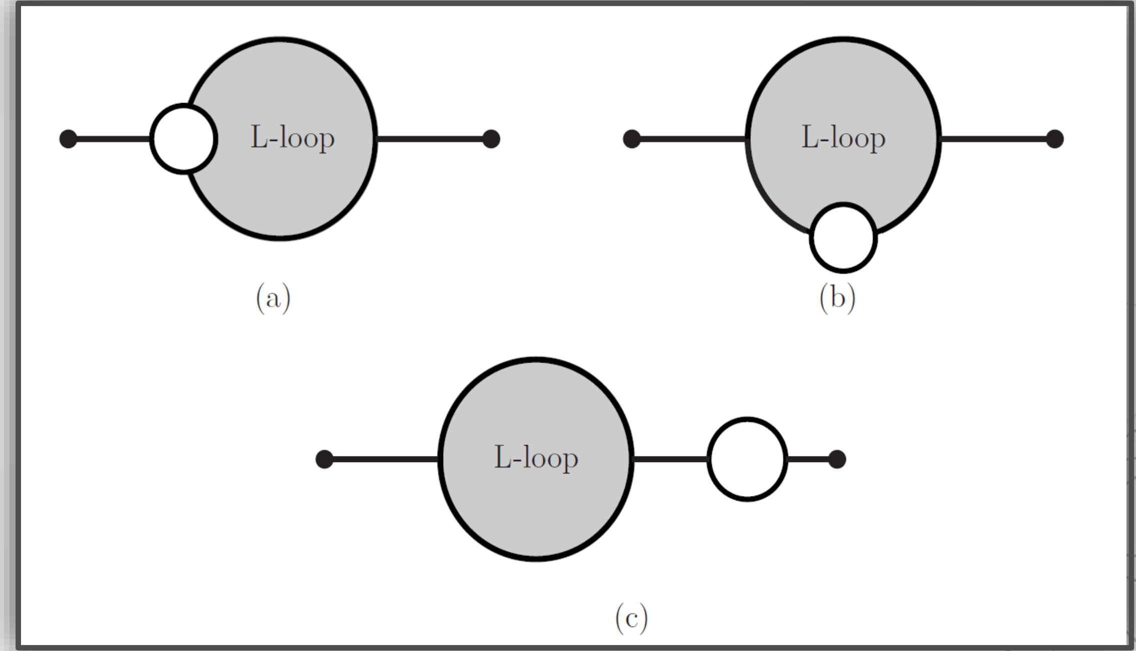
05

Higher Loop Calculation

Existance of a cut-off at all orders and Suppression of higher order corrections

Making a general higher loop diagram

An $L+1$-loop diagram can be constructed by inserting a new loop either into a line (dashed or solid line) or a vertex of a given L-loop diagram.



A nice Argument!

all vertices are reachable from an external leg with a series of head-to-tail propagators

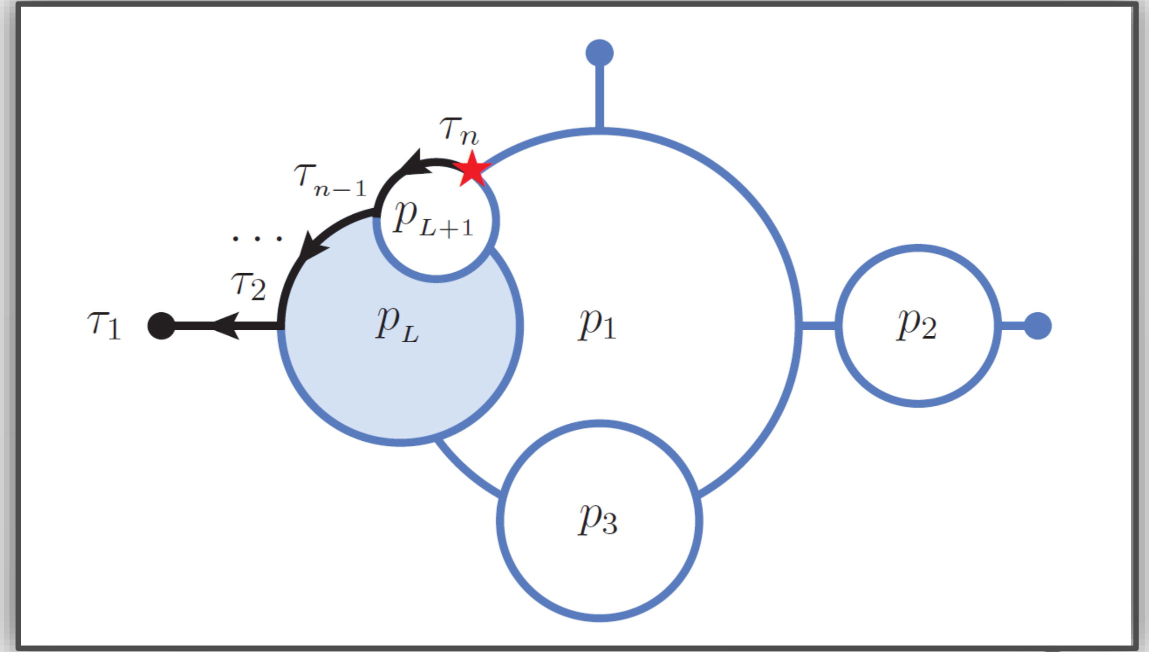
$$\tau_1 \geq \tau_2 \geq \cdots \geq \tau_{n-1} \geq \tau_n .$$

there is a net energy transfer at each vertex K_t from and to the background so that at

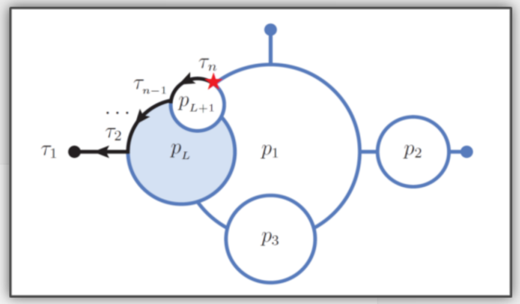
$$K_t \tau_s = \pm\omega / H$$

at we have a saddle point which gives the dominant contribution to the time integrals.

$$|K_{t_L}| \leq \cdots \leq |K_{t_2}| \lesssim \alpha\, k_{\text{ext.}} .$$



Suppression of Higher Loop Diagrams



For a generic L-loop E-point function amplitude we find

$$P_{E,L} \sim \left(\frac{\mathcal{G}^2 f^2}{H^2\alpha}\right)^{L+\frac{E}{2}-1} F_{E,L}(\alpha)$$

The first factor is due to V couplings and time integrals . $F_{E,L}(\alpha)$ is due to momentum integrals.

For any diagram with L loops, the superficial degree of UV divergence is $D = 3L - P = 3 - E$.

Therefore

$$P_{E,L} \sim \left(\frac{\mathcal{G}^2 f^2}{H^2\alpha}\right)^{L+\frac{E}{2}-1} \alpha^{N_{E,L}}, \qquad N_{E,L} \leq D = 3 - E.$$

This justifies the validity of perturbation theory when $\mathcal{G}^2 f^2/H^2\alpha \ll 1$.

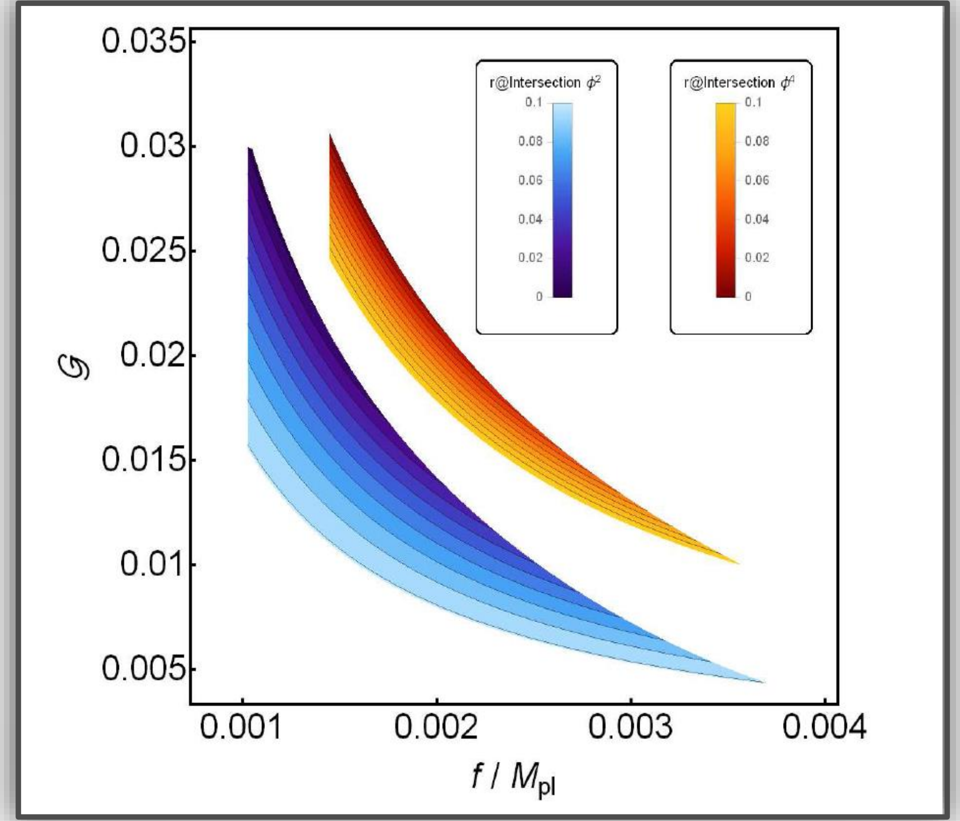
06

Observable of Model

Tensor-to-Scalar ratio and non-Gaussianities

OUR NUMBERS

Range of parameter allowed by the current observations for ϕ^2 and ϕ^4 inflationary models. Contour are values r for r<0.1. We have assumed $30 < \alpha < 125$.



Sizable NG with a New Shape

Bispectrum of scalar perturbations

$$B_\varphi(\boldsymbol{k}_1, \boldsymbol{k}_2, \boldsymbol{k}_3) := \left(\frac{5}{3}\frac{\dot{\phi}}{H}\right)^3 \frac{6A_s^2 f_{NL}^{\text{res.}}}{(k_1\, k_2\, k_3)^2} S(k_1, k_2, k_3).$$

where shape function is

$$S(k_1, k_2, k_3) := \frac{\sin\Phi(2k_1, \alpha) + \sin\Phi(2k_2, \alpha) + \sin\Phi(2k_3, \alpha)}{3},$$

and

$$\boxed{f_{NL}^{\text{res.}} = 640\sqrt{2\pi}\,\frac{\ln\alpha}{\mathcal{G}\,\alpha^{3/2}}.}$$

For typical parameters $\alpha = 100$ and $\mathcal{G} = 0.03$ we get $f_{NL} \simeq 250$. This shows that this model predicts a not so small non-Gaussianity with a shape which is somewhat supported by the data.

Summary and Coclusion

- Early universe that is lying at the intersection of high energy and cosmology is a fertile and active field of research today.

- we studied a specific slow-roll inflationary setup in which background inflationary trajectory is that of a usual, e.g. single field chaotic, inflationary model, while its perturbation theory is dominated by 1-loop correction.

- This model can reconcile convex models of inflation with Planck data.

- The diagrammatic approach presented here is a useful tool for systematic calculation of the loop corrections in cosmological correlation functions.

THANKS