Gravitational waves and BMS symmetries

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Based on arXiv:1912.03164 with *Geoffrey Compère* and *Roberto Oliveri*

27th IPM Physics Spring Conference
5 Tir 1399

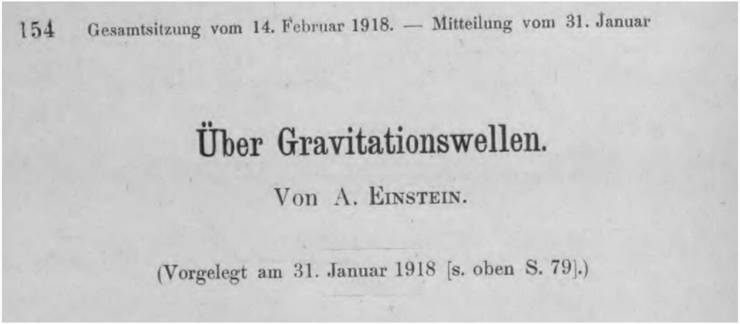
Overview

1. **A brief intro on GW discovery**
2. **Symmetries and conservation laws in GW**
3. **BMS Symmetries**

A brief intro on GW discovery

Prediction of gravitational waves

In 1918, Einstein predicted gravitational waves (GW)



154 Gesamtsitzung vom 14. Februar 1918. — Mitteilung vom 31. Januar

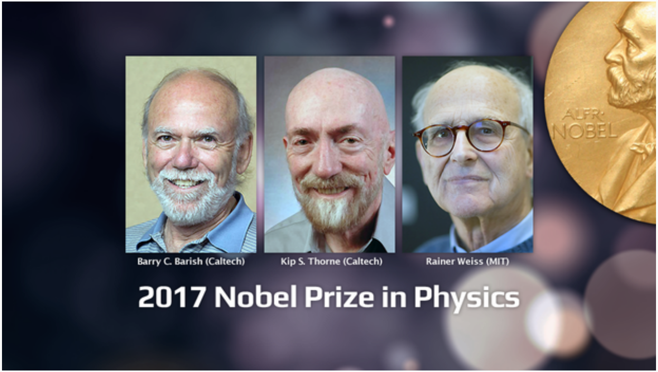
Über Gravitationswellen.

Von A. Einstein.

(Vorgelegt am 31. Januar 1918 [s. oben S. 79].)

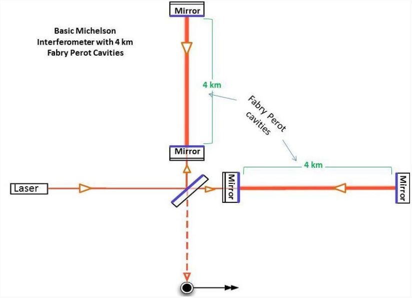
Discovery of gravitational waves

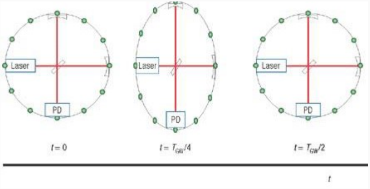
In Sep. 2015, first detection of GW by LIGO (after > 20 years of work)



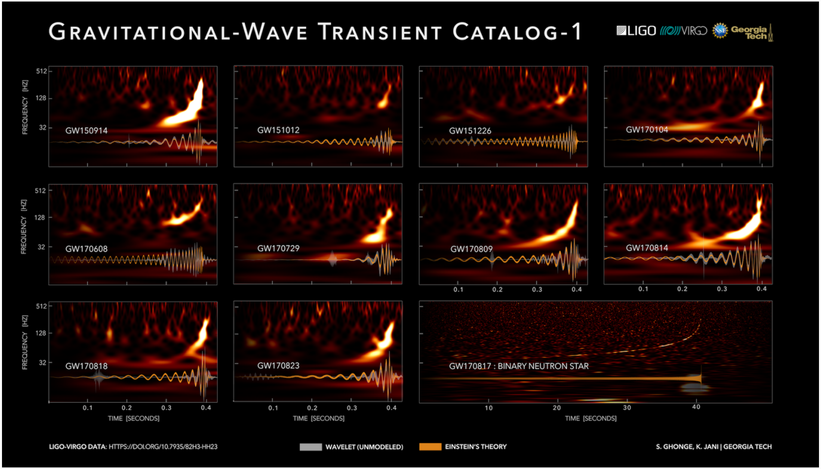
Detection of gravitational waves

LIGO-Virgo collaboration





Detection of gravitational waves



Gravity in the strong regieme

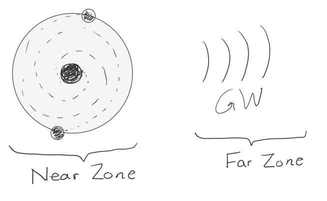
- Einstein equation: Nonlinear system of PDE
- Describing interaction between matter and spacetime
- This can be broken (perturbatively) into two sub-problems

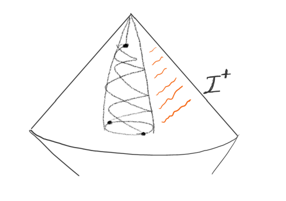
1. **GW generation** Given a source, find the gravitational field it produces
   - Post-Newtonian formalism: $v/c \ll 1$
   - Gravitational self force & Extreme mass ratio inspirals: $m/M \ll 1$
   - Numerical Relativity
2. **Back-reaction** of gravitational field on the source

Use conservation laws to study radiation reaction

Symmetries and conservation laws in GW

Asymptotic Symmetries





- Far from the source, the spacetime is asymptotically flat
- Poincaré symmetries are asymptotically restored
- One can apply Noether's theorem and associate charges to the system

$$\mathcal{E}, \qquad \mathcal{P}_i, \qquad \mathcal{J}_i, \qquad \mathcal{K}_i$$

- These charges are not conserved, but they obey flux-balance laws:

$$\boxed{\frac{dQ}{dt} = -\mathcal{F}_Q}$$

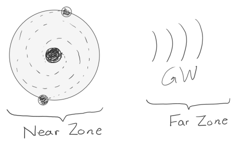
Why balance equations? [Peters, Mathews '63]

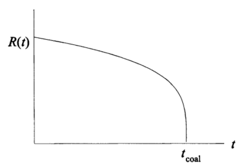
- Newtonian gravity $\mathcal{O}(v/c)^0$ No radiation. Conservation laws enough to solve Kepler problem
- First PN approximation Balance equations replace conservation laws due to radiation

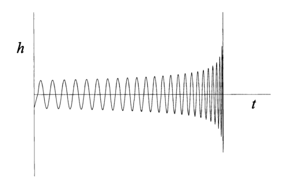
$$\frac{dE_{mech}}{dt} = -\mathcal{F}_E$$

$$E_{mech} = -\frac{Gm_1m_2}{2r}, \qquad \mathcal{F}_E = \frac{G}{5c^5}\dddot{I}_{ij}\dddot{I}_{ij}, \qquad I_{ij} = \int d^3x\rho\, x_i\, x_j$$

- Using these in balance equation determines the orbit [*Pictures: Maggiore*]







Poincare flux balance laws

Einstein quadrupole formula [*Einstein '18*]

$$\dot{\mathcal{E}} = -\frac{1}{5}\frac{G}{c^5}\dddot{I}_{ij}\dddot{I}_{ij} + \mathcal{O}(1/c^7)$$

Angular momentum flux [*Epstein, Wagoner '75, Thorne '80*]

$$\dot{\mathcal{J}}_i = -\frac{2}{5}\frac{G}{c^5}\epsilon_{ijk}\ddot{I}_{jl}\dddot{I}_{kl} + \mathcal{O}(1/c^7)$$

Linear momentum flux [*Thorne '80*]

$$\dot{\mathcal{P}}_i = -\frac{2}{63}\frac{G}{c^7}\overset{(4)}{I}_{ijk}\overset{(3)}{I}_{jk} + \mathcal{O}(1/c^9)$$

Center of mass flux [*Kozameh '16, Blanchet,Faye '18*]

$$\dot{\mathcal{G}}_i = \mathcal{P}_i - \frac{1}{21}\frac{G}{c^7}\left(\overset{(3)}{I}_{jk}\overset{(3)}{I}_{ijk} - \overset{(2)}{I}_{jk}\overset{(4)}{I}_{ijk}\right) + \mathcal{O}(1/c^9)$$

Our task: Find exact equations and extend the list

BMS Symmetries

BMS symmetry group [Bondi, Van den Burg, Metzner, Sachs]

- Found originally in the 60's and recently extended [*Barnich, Compère, Campiglia, etc.*]
- An infinite dimensional extension of the Poincaré group
- Relations found with Weinberg soft theorems and memory effects [*Strominger et.al*]

Bondi asymptotic expansion

- **Setup:** Asymptotic expansion of metric in $1/r$
- Metric in Bondi gauge [retarded coordinates (u, r, θ^A)]

$$
\begin{aligned}
ds^2 = & - c^2\, du^2 - 2c\, du dr + r^2 \gamma_{AB}\, d\theta^A d\theta^B \\
& + \frac{2G}{c^2 r} m\, du^2 + \frac{1}{r}\frac{4G}{3c^2} N_A\, du d\theta^A + r\, C_{AB}\, d\theta^A d\theta^B \\
& + \dots
\end{aligned}
$$

- **Bondi data:** $m = m(u, \theta^A), N_A = N_A(u, \theta^A)$ Bondi mass, angular momentum aspect, $C_{AB}(u, \theta^A)$ Bondi shear
- **Einstein equations**

$$
\partial_u m = -\frac{c^3}{8G}\dot{C}_{AB}\dot{C}^{AB} + \frac{c^4}{4G} D_A D_B \dot{C}^{AB},
$$

BMS symmetries and charges

- The coordinate transformation $x \to x + \xi$ is a symmetry if

$$\xi = \underbrace{T(\theta)\partial_u}_{\text{supertranslation}} + \underbrace{\epsilon^{BC}\partial_B \Phi(\theta)\partial_C}_{\text{superrotation}} + \underbrace{\gamma^{BC}\partial_B \Psi(\theta)\partial_C}_{\text{superboost}}$$

- Called BMS symmetry. It contains the Poincaré algebra

	T	Φ	Ψ
$Y_{0,0}$	time translation	-	-
$Y_{1,m}$	spatial translation	rotations	boosts

BMS flux balance laws

- Corresponding BMS charges

$$\mathcal{P}_T \equiv \frac{1}{c}\oint_S T\, m\,, \qquad \textit{Supermomentum}$$

$$\mathcal{J}_\Phi \equiv \frac{1}{2}\oint_S \epsilon^{AB}\partial_B \Phi N_A, \qquad \textit{S-angular momentum}$$

$$\mathcal{K}_\Psi \equiv \frac{1}{2c}\oint_S \gamma^{AB}\partial_B \Psi N_A \qquad \textit{S-center of mass}$$

- These charges are NOT conserved but obey flux balance equations

$$\begin{aligned}\dot{P}_T &= \frac{1}{c}\oint_S T\,\dot{m}\\ &= \frac{1}{c}\oint_S T\left(-\frac{c^3}{8G}\dot{C}_{AB}\dot{C}^{AB} + \frac{c^4}{4G}D_A D_B \dot{C}^{AB}\right)\end{aligned}$$

- Similarly for superLorentz charges

Multipole expansion

BMS multipole charges

- Why multipole expansion? PN hierarchy of fluxes
- Multipole expansion of radiation field ($n_i = \frac{x_i}{r}, N_L = n_{i_1} \cdots n_{i_\ell}$)

$$C_{ij} = \sum_{\ell=2}^{+\infty} a_\ell \left(N_{L-2}\, \boldsymbol{U}_{ijL-2} - \frac{b_\ell}{c} N_{aL-2}\, \epsilon_{ab(i} \boldsymbol{V}_{j)bL-2} \right)^{TT}$$

  in terms of mass and spin radiative multipole moments.

- Relation to source multipoles (from PN formalism)

$$\boldsymbol{U}_L = \overset{(\ell)}{\boldsymbol{I}}_L + \mathcal{O}\left(\frac{G}{c^3}\right), \qquad \boldsymbol{V}_L = \overset{(\ell)}{\boldsymbol{J}}_L + \mathcal{O}\left(\frac{G}{c^3}\right)$$

Poincarè flux balance equations

- Supermomentum balance equation

$$\dot{P}_T = \frac{1}{c}\oint_S T\left(-\frac{c^3}{8G}\dot{C}_{AB}\dot{C}^{AB} + \frac{c^4}{4G}D_A D_B \dot{C}^{AB}\right)$$

- Energy $\quad T = 1$

$$\dot{\mathcal{E}} = -\sum_{\ell=2}^{+\infty}\frac{G}{c^{2\ell+1}}\mu_\ell\left\{\dot{\boldsymbol{U}}_L\dot{\boldsymbol{U}}_L + \frac{b_\ell b_\ell}{c^2}\dot{\boldsymbol{V}}_L\dot{\boldsymbol{V}}_L\right\},$$

- Leading terms

$$\dot{\mathcal{E}} = -\frac{G}{c^5}\left(\frac{1}{5}I^{(3)}_{ij}I^{(3)}_{ij}\right) - \frac{G}{c^7}\left(\frac{1}{189}I^{(4)}_{ijk}I^{(4)}_{ijk} + \frac{16}{45}J^{(3)}_{ij}J^{(3)}_{ij}\right) + O(c^{-9})$$

- Perfect match with the literature [*Thorne '80, Blanchet, Faye '18*]
- Similar results for linear momentum and center of mass [*Kozameh '16*]

BMS balance laws

- Proper BMS balance laws [*Compère, Oliveri, AS '19, see also Nichols '18*]

$$T(\theta^A) = \boldsymbol{T}_L N_L \qquad \ell \geq 2$$

- New equations with the same PN order as the energy and angular momentum
- Example: Octupolar super angular momentum

$$\dot{\mathcal{J}}_{ijk} - \frac{2}{7c^2}\, u\, \dot{\boldsymbol{V}}_{ijk} = -\frac{G}{c^5}\left(\frac{6}{35}\,\epsilon_{pq\langle i}\,\dot{\boldsymbol{U}}_{j|p|}\boldsymbol{U}_{k\rangle q}\right) + \mathcal{O}(c^{-7})$$

Conclusion

Summary

- New flux balance equations
- In principle relevant for radiation reaction forces
- specifies gravitational memory effect

Outlook

- Wrtie BMS charges in terms of source variables
- New differential equations for source parameters

Thank you very much