Chiral Magnetic Effect in LHC

Speaker

Seyed Farid Taghavi

Institute for Research for Fundamental Science
School of Particle and Accelerators

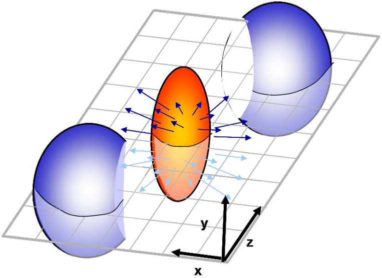
One Day Workshop on LHC Physics with Emphasis on Higgs

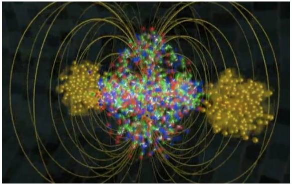
July 11 2012

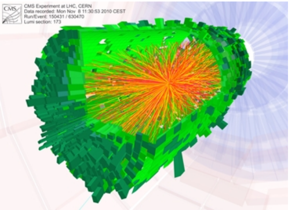
Outline

1. Motivation
2. Topological Charges
   - WKB, Tunnelling and Path Integral
   - Solitons
   - Gauge Theories and Instantons
3. Chiral Magnetic Effect
   - Axial Anomaly
   - Currents Induced by Magnetic Field
4. Summary

Motivation



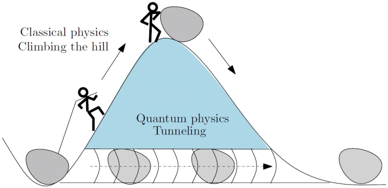




Topological charges can be seen in the Heavy Ion Collision experiment.

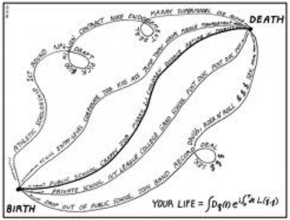
Topological Charges

WKB, Tunnelling and Path Integral



WKB appr. for tunnelling

$$\Gamma = \exp\left[-\frac{1}{\hbar}\int_{x_i}^{x_f} dx\,\sqrt{2(V-E)}\right]$$



The Path Integral Formulation of Your Life

$$x(t) = \bar{x} + \delta x$$

Path Integral in QM

$$\langle x_f | e^{-\frac{iHt}{\hbar}} | x_i \rangle = \int_{x_i}^{x_f} \mathcal{D}x \; e^{iS/\hbar}$$

$$S = \int_{-t/2}^{t/2} dt' \left[\frac{1}{2} \left(\frac{dx}{dt'} \right)^2 - V(x) \right]$$

Wick rotation $t' \to -it'$,

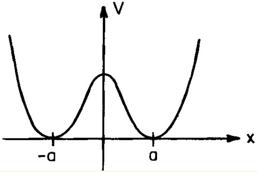
$$\langle x_f | e^{-\frac{HT}{\hbar}} | x_i \rangle = \int_{x_i}^{x_f} \mathcal{D}x \; e^{-S_E/\hbar}$$

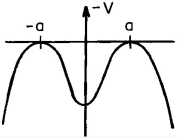
$$S_E = \int_{-T/2}^{T/2} dt \left[\frac{1}{2} \left(\frac{dx}{dt'} \right)^2 + V(x) \right]$$

Semi-Classical Limit

$$S_E(\bar{x}+\delta x) \simeq S_E(\bar{x}) + \delta x \frac{\delta S_E}{\delta x}|_{x=\bar{x}} + \delta x \frac{\delta^2 S_E}{\delta x^2}|_{x=\bar{x}} \delta x + \cdots$$

$$\int_{x_i}^{x_f} \mathcal{D}x \, e^{-S_E(x)/\hbar} \approx \mathcal{N} e^{-S_E(\bar{x})/\hbar}, \quad E = \frac{1}{2}\left(\frac{d\bar{x}}{dt'}\right)^2 - V(\bar{x})$$

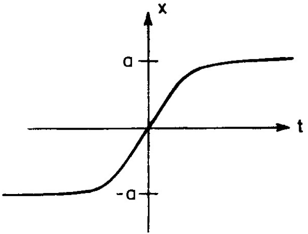




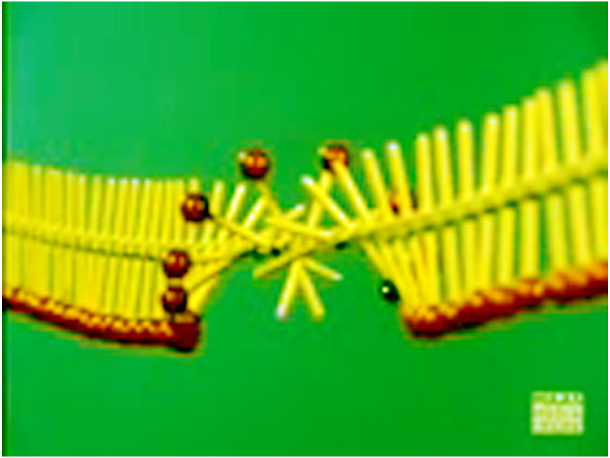
$$\langle x_f | e^{-\frac{HT}{\hbar}} | x_i \rangle \sim e^{-S_E(x)/\hbar} = e^{\frac{1}{\hbar}\int_{x_i}^{x_f} dx \sqrt{2V}}$$

In double well solution at $t_E \to \pm\infty$, x goes to $\pm a$. Therefore $V(x) \sim \omega^2 (x \mp a)^2$ where $\omega = V''(x)$. The asymptotic answer is

$$dx/dt = \sqrt{2V(x)} \to (x \mp a) \propto e^{\mp\omega t} (t_E \to \pm\infty)$$

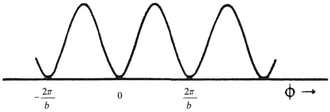


Simple Example in 1+1 D Field Theory: Solitons



$$\mathcal{L} = \frac{1}{2}\left(\frac{\partial\phi}{\partial t}\right)^2 - \frac{1}{2}\left(\frac{\partial\phi}{\partial x}\right)^2 - V(\phi), (\phi) = \frac{1}{b^2}\left(1 - \cos(b\phi)\right),$$

$$E.O.M.: \quad \frac{\partial^2\phi}{\partial t^2} - \frac{\partial^2\phi}{\partial x^2} + \frac{1}{b^2}\sin(b\phi) = 0$$



Infinite number constant solutions with zero energy:

$$\phi = \frac{2\pi n}{b}, \quad n = 0, \pm 1, \pm 2, \ldots$$

and also one-soliton answer

$$\phi(x,t) = \frac{4}{b} arctan(e^{\pm(\gamma(x-vt)/\sqrt{b}}), \quad \gamma = \frac{1}{\sqrt{1-v^2}}$$

$$\phi(-\infty) = 0, \quad \phi(\infty) = 2\pi/b$$

$$M \equiv E = 8/b^2$$

Movie!

What kind of conserved current?!

$$J^\mu = \frac{b}{2\pi}\epsilon^{\mu\nu}\partial_\nu\phi$$

What kind of conserved charge?!

$$\begin{aligned} Q &= \int_{-\infty}^{\infty} J^0 dx \\ &= \frac{b}{2\pi}\int_{-\infty}^{\infty}\frac{\partial\phi}{\partial x}dx \\ &= \frac{b}{2\pi}[\phi(\infty)-\phi(-\infty)] = N, \quad N = 0, \pm 1, \pm 2, \ldots \end{aligned}$$

Gauge Theories and Instantons

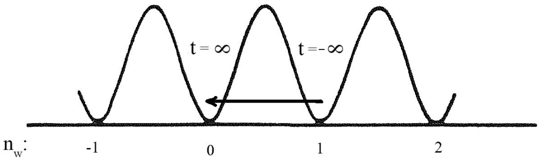
Non-abelian gauge theory action

$$S = \int d^4x \left[-\frac{1}{4} \mathcal{F}_{a\mu\nu} \mathcal{F}_a^{\mu\nu} \right]$$

$$\mathcal{F}_a^{\mu\nu} = \partial^\mu A_a^\nu - \partial^\nu A_a^\mu + g f_{abc} A_b^\mu A_c^\nu$$

conserved current

$$K^\mu = \epsilon^{\mu\nu\rho\sigma} A_{a\nu} \left[\mathcal{F}_{a\rho\sigma} - \frac{g}{3} f_{abc} A_{b\rho} A_{c\sigma} \right]$$



Winding Number

$$\int d^3x K^0 = 24\pi^2 n_w(t_E = \pm\infty), \quad n_w = 0, \pm 1, \pm 2, \ldots$$

$$Q_w = \frac{1}{32\pi^2} \int d^4x \mathcal{F}_{a\mu\nu} \tilde{\mathcal{F}}_a^{\mu\nu}$$

$$Q_w = n_w(t_E = \infty) - n_w(t_E = -\infty), \quad \tilde{\mathcal{F}}_a^{\mu\nu} = \frac{1}{2}\epsilon_{\mu\nu\rho\sigma}\mathcal{F}_a^{\rho\sigma}$$

Chiral Magnetic Effect

• $D.E.Kharzeev, Phys.Lett.B633(2006)260.$
• $K.Fukushima, D.E.Kharzeev, H.J.Warringa, Phys.Rev.D78 : 074033, 2008.$
• $D.E.Kharzeev, L.D.McLerran, H.J.Warringa, Nucl.Phys.A803 : 227 - 253, 2008.$
• $D.E.Kharzeev, Ann.Phys.325(2010)205 - 218.$

Axial Anomaly

QCD Lagrangian

$$\mathcal{L} = -\frac{1}{4}\mathcal{F}_{a\mu\nu}\mathcal{F}_a^{\mu\nu} + \bar{\psi}\,[i\gamma^\mu(\partial_\mu - igA_{a\mu}t_a)\,]\psi$$

Global Symmetries

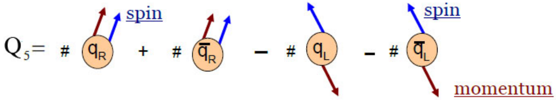
$$\psi \to e^{i\alpha}\psi, \qquad j^\mu(x) = \bar{\psi}(x)\gamma^\mu\psi(x), \quad \partial_\mu j^\mu = 0$$

$$\psi \to e^{i\alpha\gamma^5}\psi, \qquad j^{\mu 5}(x) = \bar{\psi}(x)\gamma^\mu\gamma^5\psi(x), \quad \partial_\mu j^{\mu 5} \neq 0(Anomaly!)$$

$$\partial_\mu j^{\mu 5} = -\frac{g^2}{16\pi^2}\mathcal{F}_{a\mu\nu}\tilde{\mathcal{F}}_a^{\mu\nu}$$

Currents Induced by Magnetic Field

The charge correspond to $\partial_\mu j^{\mu 5}$ can be written as follows [1]

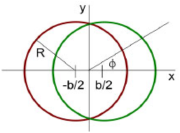


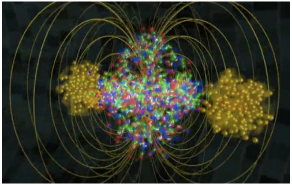
$$-2Q_w = -\frac{1}{16\pi^2}\int d^4x \mathcal{F}_{a\mu\nu}\tilde{\mathcal{F}}_a^{\mu\nu} = \int d^4x \partial_\mu j^{\mu 5}$$
$$= \int_{-\infty}^{\infty} dt \frac{dQ_5}{dt} = Q_5(t=\infty) - Q_5(t=-\infty)$$

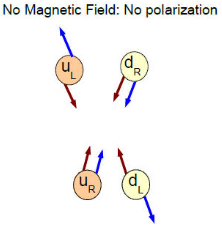
Assuming at $t \to -\infty$, we have $N_L = N_R$ therefore:

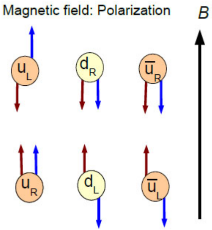
$$(N_L - N_R)_{t=\infty} = 2Q_W$$

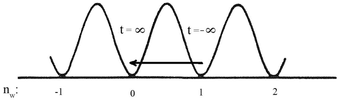
[1] In the rest of slides pictures of Kharzeev and Warringa presentations frequently used.

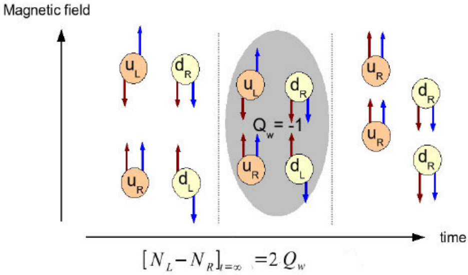

y
R
ϕ
x
-b/2
b/2




No Magnetic Field: No polarization
u_L
d_R
u_R
d_L


Magnetic field: Polarization
B
u_L
d_R
$\bar{u}_R$
u_R
d_L
$\bar{u}_L$

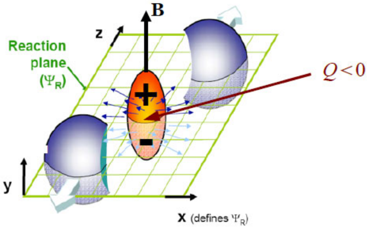




In the presence of the topological charges an electric current induced by magnetic field.

$$J = \sigma_B B$$

Charge Separation



Thank You