Magnetic design for electron Linac

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Magnet Categories

- Electromagnets
  - Normal conducting
    - Magnetic field up to 2 T combination of coil and yoke
  - Superconducting
    - Saturation effect in iron
    - Max. current density in copper 100 A/mm²

- Permanent magnets
  - Constant magnetic field
  - Savings
  - Sm-Co, Neodymium
Why do we need magnets?
Dipole magnets

Guide the beam to keep it on the orbit

- Magnetic flux density: $B_x = 0; B_y = \text{const}$, $NI = Bh/\mu_0$

<table>
<thead>
<tr>
<th>Type</th>
<th>Accessibility</th>
<th>Iron (weight)</th>
<th>Mechanical stability</th>
<th>Field quality</th>
<th>Shims required</th>
</tr>
</thead>
<tbody>
<tr>
<td>C-magnet</td>
<td>very good</td>
<td>high</td>
<td>poor</td>
<td>asymmetric</td>
<td>yes</td>
</tr>
<tr>
<td>H-magnet</td>
<td>poor</td>
<td>medium</td>
<td>good</td>
<td>symmetric</td>
<td>yes</td>
</tr>
<tr>
<td>O-magnet</td>
<td>very poor</td>
<td>low/medium</td>
<td>good</td>
<td>symmetric</td>
<td>no</td>
</tr>
</tbody>
</table>
Quadrupoles

- Focus the beam
  - Magnetic flux density: \( B_x = B_2 y; \ B_y = B_2 x, \ NI = B'r^2/\mu_0 \)

- Very limited space for coils
- Maximum space for coils
- Saturation around the region of the pole roots
- Mechanically less stable
- More complicated to produce
- More expensive.
Solenoids

◆ Weak focusing
Magnetic flux density: \( B_r \approx (r/2) \left[ dB_z(0,z)/dz \right] \), \( B_z(z=0) \approx 0.5NI/ (a^2+b^2)^{0.5} \)
✓ Used in experiments or low-energy beam lines

\[
H_z(z) = \frac{NI}{4b} \left\{ \frac{b-z}{a_2-a_1} \ln \left\{ \frac{(a_2 + (a_2^2 + (b-z)^2)^{1/2})}{a_1 + (a_1^2 + (b-z)^2)^{1/2}} \right\} \right. \\
\left. + \frac{b+z}{a_2-a_1} \ln \left\{ \frac{(a_2 + (a_2^2 + (b+z)^2)^{1/2})}{a_1 + (a_1^2 + (b+z)^2)^{1/2}} \right\} \right\}
\]

• Off axis particles: \( v_z \times B_r = F_\theta \)
• \( v_\theta \times B_z = F_r \approx r \)
◆ Sextupoles: correct chromatic aberrations
✓ Magnetic flux density: \( B_y = B_3(x^2 - y^2) \), \( NI = B''r^3/3\mu_0 \)

◆ Corrector: correct horizontal or vertical beam displacement

◆ Kicker & Septum: used for injection and extraction
Design requirements

- Field strength (gradient) and magnetic length
- Integrated field strength (gradient)
- Aperture and good field region
- Operation mode: continuous, cycled
- Field quality:
  ✓ field homogeneity
  ✓ maximum allowed multipole errors
- **Cooling**

\[ \alpha = \frac{\int B ds}{B_0 \rho_0} \]
Conductor

For a given field we need a certain current

• Can be approximated

• Final values from computer codes FEMM, POISSON, Opera, Comsol, TOSCA, etc.
Material Yoke

- Affected by their purity, the metalworking processes (hot and cold working, subsequent annealing), and the resulting microstructure.

- **Low-carbon steels**: frequently used, including in magnet construction (steel 1010)

- **Silicon steels**: allows for an increase in permeability and decrease in hysteresis loss. Also thanks to additions of Al and Mn, eddy current losses decrease due to higher resistivity.

- **Fe-Ni alloys**: used for shielding sensitive electronic equipment against static or low-frequency magnetic field
# Coil cooling

<table>
<thead>
<tr>
<th>Current density (A/mm²)</th>
<th>Cooling</th>
</tr>
</thead>
<tbody>
<tr>
<td>≤ 2A/mm²</td>
<td>Air</td>
</tr>
<tr>
<td>≤ 3A/mm²</td>
<td>Tap water</td>
</tr>
<tr>
<td>≤ 10A/mm²</td>
<td>Demineralized water</td>
</tr>
</tbody>
</table>

\[
u_{avg} \approx 0.3926 \, d^{0.714} \left( \frac{\Delta p}{l} \right)^{0.571}
\]

\[
\Delta T = 304 \frac{p}{u_{avg} \, d^2} \times 10^{-9}
\]

\[
R_e = \frac{u_{avg} \, d}{\nu}
\]

- The velocity of the cooling medium should be **sufficiently high** to guarantee a turbulent flow **but low enough** (\(u_{avg} \leq 5 \text{ m/s}\)) to avoid erosion and vibration.

- **The flow is turbulent** → **Reynolds number > 4000**

- \(l, d\): cooling circuit length and diameter
- \(\Delta p\): pressure drop
- \(P\): power
- \(\Delta T\): temperature rise
Multipole errors

\[ B_y + iB_x = \sum_{n=1}^{\infty} C_n \left( \frac{x + iy}{R_{\text{ref}}} \right)^{n-1} \]

\[ C_n = |C_n| e^{i n \varphi_n} = B_n + iA_n \]

- \( C_n \): multipole field of order \( n \)
- \( B_n, A_n \): Absolute normal and skew harmonics
- \( R_{\text{ref}} \): typically \( 2/3 \) of the yoke aperture
- \( \varphi \): angular orientation of the magnet around \( z \) axis

\[ c_n = b_n + i a_n = \frac{C_n}{B_N} \]

- \( B_N \): main field
- \( b_n, a_n \): Normalized normal and skew harmonics in units of \( 10^{-4} \)
## Multipole errors

<table>
<thead>
<tr>
<th>Error</th>
<th>Multipoles</th>
</tr>
</thead>
<tbody>
<tr>
<td>Designed magnet</td>
<td>Odd multipoles of the main field</td>
</tr>
<tr>
<td>Horizontal offset</td>
<td>$a_2, a_4, a_6, \ldots$</td>
</tr>
<tr>
<td>Vertical offset</td>
<td>$b_2, b_4, b_6, \ldots$</td>
</tr>
<tr>
<td>Rotation between two halves</td>
<td>$b_1, b_3, b_5, b_7, \ldots$</td>
</tr>
<tr>
<td>Differences in the lengths</td>
<td>$a_1, a_3, a_5, a_7, \ldots$</td>
</tr>
</tbody>
</table>
Hollow solenoid

- Minimum $B_z=800$ Gauss

Internal diameter=25 cm

Coil width and height=9 mm

d_pipe= 4.48 mm
Buncher and first accelerating tube

- $n_1=7$, $n_2=11$, $I=300$ A

**First accelerating tube**
- $L_2=0.5$ m

**Buncher**
- $L_1=0.2$ m
Solenoid cooling

- **$P$ (W)**: 572
- **$\Delta T$**: 3.33
- **$Q$ (LIT/MIN)**: 2.46
- **Flow velocity (m/s)**: 2.61
- **Re**: 11852.2
- **$\Delta P$ (Pascal)**: 250000
## Spectrometer Dipole

### Requirements and constraints

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Nominal magnetic flux density</td>
<td>0.1</td>
<td>T</td>
</tr>
<tr>
<td>Bending angle</td>
<td>25</td>
<td>Degree</td>
</tr>
<tr>
<td>Maximum total gap height (not including shims)</td>
<td>40</td>
<td>mm</td>
</tr>
<tr>
<td>Horizontal good field region</td>
<td>±20</td>
<td>mm</td>
</tr>
<tr>
<td>Vertical good field region</td>
<td>±10</td>
<td>mm</td>
</tr>
<tr>
<td>Integrated field homogeneity</td>
<td>( \leq \pm 5 \times 10^{-3} )</td>
<td></td>
</tr>
</tbody>
</table>

- C-type magnet
- Racetrack coils
- Straight
- Low carbon steel 1010
Coil parameters

- $NI = Bg/\mu_0 = 3183$ A, $I = 4$ A, $N = 400$
- $J = I/A = 1.17$ A/mm²
- $R_{magnet} = \rho l/A = 2.58$ Ω
- $P = \rho l/A.I^2 = 20.6$ W
- $\Delta T/\Delta t = P/m_{coil} \cdot C_{cu} = 0.007$ K/s to reach a $\Delta T = 15$ K will require a powering time of $\Delta t \approx 33$ min without cooling

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Unit</th>
<th>Remark</th>
</tr>
</thead>
<tbody>
<tr>
<td>Type</td>
<td></td>
<td></td>
<td>Racetrack</td>
</tr>
<tr>
<td>Cooling</td>
<td></td>
<td></td>
<td>Air-cooled</td>
</tr>
<tr>
<td>Conductor material</td>
<td></td>
<td></td>
<td>Copper</td>
</tr>
<tr>
<td>Conductor dimensions</td>
<td>1.85 x 1.85</td>
<td>mm x mm</td>
<td></td>
</tr>
<tr>
<td>Cross section of conductor</td>
<td>3.42</td>
<td>mm²</td>
<td></td>
</tr>
<tr>
<td>Coil windings</td>
<td>25 x 16 = 400</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Coil cross-section</td>
<td>46.25 x 29.6</td>
<td>mm x mm</td>
<td></td>
</tr>
</tbody>
</table>
To define its main parameters like the pole width, shims at the extremities of the pole, and coil position.
Field quality

With Shim  Without Shim

Harmonic order

Unit

Unit

x(cm)

With Shim  Without Shim

Field quality

21

Uniformity(%)
3D simulation by Comsol

- $\Delta jB_y$ of $0.4 \times 10^{-3}$
- $\Delta jB_y$ of $0.8 \times 10^{-3}$
Solenoid for spectrometer system

\[ \int B_z^2 \, dz = 0.015 \, T^2 \, m; \quad L = 12 \, cm \]

\[ I = 10 \, A; \quad \text{coil area} = 1.85 \times 1.85 \, \text{mm}^2; \quad J = 2.9 \, A/ \, \text{mm}^2; \quad N = 65 \times 59; \quad a_1 = 2.5 \, cm; \quad a_2 = 13.4 \, cm \]
\[ \Rightarrow \int B_z^2 \, dz = 0.0087 \, T^2 \, m, \quad \text{mass} = 58.5 \, kg \]

After shielding \[ \Rightarrow \int B_z^2 \, dz = 0.0149 \, T^2 \, m \]
- $d_2 = 1\, \text{cm}$
- $d_3 = 0.3\, \text{cm}$
- $d_4 = 0.3\, \text{cm}$
- $d_5 = 1\, \text{cm}$
Printed circuit steerer

Why PC steerer:
- Very small magnetic field (about 6 G at the aperture)
- Iron loaded magnets: hysteresis, remnant field and cross coupling of steerers for orthogonal planes are difficult to handle at the low field strength \( \rightarrow \) iron free air coils
- Cost and small aspect ratio and ease of manufacturing
- Integrated field strength: 27.25 G.cm @ 2 A
- 18 \( \mu \)m thick copper layer, 725 \( \mu \)m minimal gap between two conductors, 600 \( \mu \)m minimal width of the conductor with 11 turns, 2.264 cm radius, length of 3.345 cm.
The current density on a cylindrical shell around the beam axis has to be proportional to the cosine of the azimuth angle $\theta$.

$$C_m(z) = \frac{1}{c} I^{16} P_m \frac{(2m - 1)!!}{(m - 1)! 2^{2m}} R^m \sum_{\varphi_i} \cos m \varphi_i [G_m(z + l_i) - G_m(z - l_i)]$$

$$G_m(t) = \frac{1}{m} \frac{t}{(R^2 + t^2)^{m+1/2}} + \int_0^t \frac{dt}{(R^2 + \tau^2)^{m+1/2}}$$
Comsol simulation
The measurement method is chosen based on gap size, field strength and homogeneity, local vs. integral field, bandwidth and the required accuracy, the availability of equipment and resources.

- **Nuclear magnetic resonance**
  - Accuracy of 1 ppm.
  - High precision of field mapping, calibration tool.

- **Hall generators**
  - The most serious limit on the accuracy:
  - Offset drifts with temperature, Non-linearity of the calibration curve.

- **Induction coil**
  - Based on Faraday’s induction law
  - High accuracy to higher harmonics.

- **Stretched wire**
  - Very high absolute accuracy
  - Only integral measurement.
Hall probe system at IPM

- Active area 150 µm* 150 µm
- Resolution 1 µT

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Specification</th>
</tr>
</thead>
<tbody>
<tr>
<td>Full scale (nominal)</td>
<td>2 T</td>
</tr>
<tr>
<td>Sensitivity to D.C magnetic field</td>
<td>5 V/T</td>
</tr>
<tr>
<td>Tolerances of sensitivity (B = 1T, D.C)</td>
<td>0.03%</td>
</tr>
<tr>
<td>Temperature coefficient of sensitivity</td>
<td>&lt;25 ppm/°C</td>
</tr>
<tr>
<td>Residual non-linearity (up to 2 T)</td>
<td>&lt;0.05%</td>
</tr>
<tr>
<td>Planar Hall effect: Vplan /Vvert (1 T)</td>
<td>&lt;0.01%</td>
</tr>
<tr>
<td>Offset (B = 0 T)</td>
<td>&lt;± 1 mV (± 0.2 mT)</td>
</tr>
<tr>
<td>Long term instability of sensitivity</td>
<td>&lt; 1% over 10 years</td>
</tr>
</tbody>
</table>
The flux corresponding to a given coil position can be obtained by flipping or rotating the coil, pulsing the field from zero.

**Advantages**

- High accuracy to higher harmonics
- A linear sensor
- Circular apertures

\[
C_n = \frac{2\Psi_{n+1}R_{ref}^{n-1}}{Pk_n}
\]

\[
k_n = \frac{N_t L_c}{n} \left[ \left( R_0 + i \frac{W_{eff}}{2} e^{i\alpha} \right)^n - \left( R_0 - i \frac{W_{eff}}{2} e^{i\alpha} \right)^n \right] e^{i\varphi_0}
\]

- \( R_0 \) : average radius
- \( W_{eff} \) : effective width of the coil
- \( \varphi_0, \alpha \) : phase and the tilt angle of the coil
Thanks for your attention