From conformal invariance to local dynamical scaling:
applications to the non-equilibrium growth of interfaces

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The kinetics of growing interfaces has a natural dynamical scaling at late times. Therefore, and without having to fine-tune any physical parameter, the behaviour of dynamical interfaces can be cast into dynamical universality classes. These can be distinguished through the values of several exponents, namely the dynamical exponent z, the growth exponent β and the roughness exponent α. Analysis of the long-time relaxation permits a finer distinction through the values of the ageing exponents a, b and the autocorrelation exponent λ_C and the autoresponse exponent λ_R. Hence, dynamical scaling can be used to define scaling functions for surface roughness and height correlations and responses. These scaling functions are *universal* in the sense that their form does not depend on the 'fine details' of the formulation of the underlying models. Recent considerable progress which now allows to discuss the universality of these exponents, based on experiments, will also be mentioned.

Our main question will be: having identified universal scaling functions in a clear physical context, what can be said about the *form* of these scaling function ?

A conceivable mechanism to find universal scaling function is provided by a dynamical symmetry. The best-studied are example are equilibrium critical phenomena, where global scale-invariance is extended to *conformal invariance* wherein the rescaling factors can depend on the space position, but such that angles are kept unchanged. In interface growth, the most simple equation of motion is the one of the Edwards-Wilkinson universality class, which reduces to a simple noisy diffusion equation. It turns out that the analysis of this problem can be separated into two steps: first, one finds all dynamical symmetries of the simple noise-less diffusion equation. The sought-for symmetry Lie group is known as the *Schrödinger group*. Second, one uses the Bargman superselection rules obtained to derive explicit reduction formulæ which express any desired average, of the full, noisy theory, in terms of 'deterministic averages' obtained when the noise is turned off. This provides the basis for the prediction of non-equilibrium responses and correlators which can be confronted with the results of specific theoretical models or with experiments.

The techniques presented are quite general and should also be useful to people working in string-theory (e.g. AdS/CFT correspondence) or conformal invariance in dimensions $d > 2$.

The following contents of the lectures is planned:

1. Growing interfaces; Family-Viscek scaling description; non-equilibrium relaxations; exponents; universality from experiments [1, 12]

2. Stochastic Langevin equations; Non-equilibrium field-theory à la Janssen-de Dominicis; response operators [15]

3. the noisy diffusion equation and its dynamical symmetries; Schrödinger algebra; Bargman superselection rule; reduction formulæ for noisy responses and correlators [2]

4. the Edwards-Wilkinson universality class: a test of Schrödinger-invariance [13]
5. breaking of time-translation-invariance and the ageing algebra; emergence of the second, independent scaling dimension of a scaling operator $\varphi(t, \boldsymbol{r})$ [2]
6. the Arcetri model and its exact solution; ageing-invariance of the Arcetri model; ageing invariance and second scaling dimension in the Glauber-Ising model [7, 8, 2]
7. causality conditions on Schrödinger- or ageing-co-variant two-point response functions
8. meta-conformal and galilean conformal invariances as variants of the standard conformal invariance at equilibrium; ballistic transport and Boltzmann equation [14, 9]
9. causality conditions in galilean conformal and meta-conformal invariances [8, 10]
10. physical ageing in the Kardar-Parisi-Zhang universality class; logarithmic ageing invariance; description of the non-equilibrium response [4, 3, 11]
11. a non-local Langevin equation: diffusion-limited erosion, vicinal surfaces and meta-conformal invariance [9]

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