IR modifications of gravity and the role of Massive gravity

Shahab Shahidi

Damghan University

Topical workshop on Physics
7-8 October 2015

Plan of the talk

- Early massive gravity,

  Fierz-Pauli, vDVZ, Vainstein, Boulware-Deser (BD) ghost.
- dRGT theory,

  Non-linear terms to avoid BD ghost?

  Self-acceleration? instability? strong coupling?
- The Quasi-Dilaton (QD) massive gravity,

  Theory and motivations, instability remains.

  Solve the problem in decoupling-limit (DL), not in full theory.
- Extended QD massive gravity?

  Seems to be healthy but not yet fully confirmed..

Introduction

- In the large scales/IR limit, deviations from GR is observed, such as the self-accelerating expansion of the Universe.
- Bunch of modified theories of gravity exists!
- In Massive gravity, one gives the graviton more d.o.f.
  - The accelerated expansion can be explained by the weakness of gravitation in IR instead of adding mysterious dark energy!
  - Theoretical interests to build a healthy theory of massive graviton.
- Consistent theory of massive gravity contains 5 dof.
- The problem is to find a consistent generally covariant theory of massive gravity.

Fierz-Pauli action

- The FP Lagrangian

$$\mathcal{L} = -h^{\mu\nu}\mathcal{E}^{\alpha\beta}_{\mu\nu}h_{\alpha\beta} - \frac{1}{2}m^2(h_{\mu\nu}h^{\mu\nu} - h^2) + \frac{2}{M_{pl}}h_{\mu\nu}T^{\mu\nu}.$$

  where $\mathcal{E}^{\alpha\beta}_{\mu\nu}$ is the Einstein operator.
- FP mass term is the unique ghost and tachyon free mass term for a spin-2 field. (M Fierz, et al. PRSLA 1939)
- Fine tuning in the mass term!

$$-\frac{1}{2}m^2\left[h_{\mu\nu}h^{\mu\nu} - (1-a)h^2\right]$$

  describe a scalar ghost with mass $m_g^2 = \frac{3-4a}{2a}m^2$.
- $m_g \to \infty$ in the limit $a \to 0$ rendering it non-dynamical.

Fierz-Pauli action

- The FP Lagrangian

$$\mathcal{L} = -h^{\mu\nu}\mathcal{E}^{\alpha\beta}_{\mu\nu}h_{\alpha\beta} - \frac{1}{2}m^2(h_{\mu\nu}h^{\mu\nu} - h^2) + \frac{2}{M_{pl}}h_{\mu\nu}T^{\mu\nu}.$$

  where $\mathcal{E}^{\alpha\beta}_{\mu\nu}$ is the Einstein operator.
- The $m = 0$ case is invariant under the linearized general coordinate transformations

$$\delta h_{\mu\nu} = \partial_\mu \xi_\nu + \partial_\nu \xi_\mu.$$

  hence, 2 d.o.f.
- The massive case breaks the above symmetry, and so acquires 5 d.o.f.

The vDVZ Problem

- The massless limit of the massive theory should reduces to the massless theory (linearized GR).
- The propagator for a massive spin-2 field is

$$G^{massive}_{\mu\nu\alpha\beta} = \frac{\tilde{\eta}_{\mu(\alpha}\tilde{\eta}_{\beta)\nu} - \frac{1}{3}\tilde{\eta}_{\mu\nu}\tilde{\eta}_{\alpha\beta}}{\Box - m^2}$$

with $\tilde{\eta}_{\alpha\beta} \equiv \eta_{\alpha\beta} + \frac{p_\alpha p_\beta}{m^2}$

- The propagator of a massless graviton is

$$G^{massless}_{\mu\nu\alpha\beta} = \frac{\eta_{\mu(\alpha}\eta_{\beta)\nu} - \frac{1}{2}\eta_{\mu\nu}\eta_{\alpha\beta}}{\Box}$$

The vDVZ Problem

- The gravitational exchange amplitude via a massive spin-2 field in the limit $m \to 0$ becomes

$$\mathcal{A}_{TT'}^{m\to 0} \to -\frac{2}{M_{pl}} \int d^4x T'^{\mu\nu} \frac{1}{\Box} \left(T_{\mu\nu} - \frac{1}{3} T \eta_{\mu\nu} \right) .$$

- The gravitational exchange amplitude via a massless graviton becomes

$$\mathcal{A}_{TT'}^{massless} \to -\frac{2}{M_{pl}} \int d^4x T'^{\mu\nu} \frac{1}{\Box} \left(T_{\mu\nu} - \frac{1}{2} T \eta_{\mu\nu} \right) .$$

- Massless case has $\frac{1}{2}$ but massive case has $\frac{1}{3}$.

(H van Dam, et al. NPB 1970; VI Zakharov, PZETF 1970)

THE vDVZ PROBLEM

- The solar system tests using massive graviton differs from the massless (GR) case:
- The bending of light at impact parameter b is
  - Massless graviton:
  $$\alpha = \frac{4GM}{b}$$
  - Massive graviton:
  $$\alpha = \frac{3GM}{b}$$
- Massive prediction differs from GR by 25%!

The Stuckelberg trick

- The mass term breaks linearized version of general covariance.
- In order to analyze dof of the theory, we have to restore the general covariance.
- Transform fields as

$$h_{\mu\nu} \to h_{\mu\nu} + \frac{2}{m}\partial_{(\mu}A_{\nu)} \quad \& \quad A_\mu \to A_\mu + \frac{1}{m}\partial_\mu \phi$$

and then

$$h_{\mu\nu} = h'_{\mu\nu} + \phi\eta_{\mu\nu},$$

Stuckelberg trick

- The general $m \neq 0$ propagators will become

$$
\begin{aligned}
h'_{\mu\nu}: &\qquad \frac{-i}{p^2+m^2}\left[\eta_{\alpha(\sigma}\eta_{\lambda)\beta}-\frac{1}{2}\eta_{\alpha\beta}\eta_{\sigma\lambda}\right], \\
A_\mu: &\qquad \frac{1}{2}\frac{-i\eta_{\mu\nu}}{p^2+m^2}, \\
\phi: &\qquad \frac{1}{6}\frac{-i}{p^2+m^2},
\end{aligned}
$$

- All are continuous in the limit $m \to 0$.

Stuckelberg trick

- This will change the $m \to 0$ limit of the action with conserved source to

$$S = \int d^4x \, \mathcal{L}_{m=0}(h') - \frac{1}{2} F_{\mu\nu} F^{\mu\nu} - 3\partial_\mu \phi \partial^\mu \phi + \kappa h'_{\mu\nu} T^{\mu\nu} + \kappa \phi T,$$

- The action respects to

$$\delta h'_{\mu\nu} = \partial_\mu \xi_\nu + \partial_\nu \xi_\mu, \; \delta A_\mu = 0$$
$$\delta A_\mu = \partial_\mu \Lambda, \quad \delta \phi = 0.$$

The massless limit contains a massless spin-2 $h_{\mu\nu}$, a massless spin-1 A_μ and a massless scalar ϕ.

Stuckelberg trick

- This will change the $m \to 0$ limit of the action with conserved source to

$$S = \int d^4x \; \mathcal{L}_{m=0}(h') - \frac{1}{2} F_{\mu\nu} F^{\mu\nu} - 3\partial_\mu \phi \partial^\mu \phi + \kappa h'_{\mu\nu} T^{\mu\nu} + \kappa \phi T,$$

- The vDVZ discontinuity is associated with the coupling of the trace of energy-momentum tensor with the scalar graviton.
- This discontinuity can be cured by non-linear interactions!

(AI Vainstein, PLB 1972)

Non-linear massive gravity

- The simplest non-linear massive gravity action is

(AI Vainstein, PLB 1972)

$$S = \frac{1}{2\kappa^2}\int d^4x \left[(\sqrt{-g}R) - \sqrt{-g^0}\frac{1}{4}m^2 g^{(0)\mu\alpha} g^{(0)\nu\beta} \left(h_{\mu\nu}h_{\alpha\beta} - h_{\mu\alpha}h_{\nu\beta}\right)\right].$$

- Unfortunately, the above action has an unhealthy sixth dof, the Boulware-Deser ghost. (DG Boulware, et al. PRD 1972)
- **The problem** is how one can add non-linear self-interaction terms to FP, making the Vainstein mechanicm works and at the same time avoids BD ghost.
- This was answered by de Rham, Gabadadze and Tolley, known as dRGT theory. (C de Rham, et al. PRL 2010)

dRGT massive gravity

- The dRGT action is (C de Rham, et al. PRL 2010)

$$
\begin{aligned}
S = & - M_p^2 \int d^4x \sqrt{-g}\, R(g) \\
& + 2M_p^2 m^2 \int d^4x \sqrt{-g} \left[\mathcal{U}_2(\mathcal{K}) + \alpha_3 \mathcal{U}_3(\mathcal{K}) + \alpha_4 \mathcal{U}_4(\mathcal{K}) \right].
\end{aligned}
$$

where

$$
\begin{aligned}
\mathcal{U}_2(\mathcal{K}) &= [\mathcal{K}]^2 - [\mathcal{K}^2]\,, \\
\mathcal{U}_3(\mathcal{K}) &= [\mathcal{K}]^3 - 3[\mathcal{K}][\mathcal{K}^2] + 2[\mathcal{K}^3]\,, \\
\mathcal{U}_4(\mathcal{K}) &= [\mathcal{K}]^4 - 6[\mathcal{K}^2][\mathcal{K}]^2 + 8[\mathcal{K}^3][\mathcal{K}] + 3[\mathcal{K}^2]^2 - 6[\mathcal{K}^4]\,,
\end{aligned}
$$

and

$$
\mathcal{K}^\mu_\nu(g, \phi^a) = \delta^\mu_\nu - \sqrt{g^{-1} f}^{\,\mu}{}_{\nu}
$$

- $f_{\mu\nu}$ is the fiducial metric: $f_{\mu\nu} = \eta_{ab}\partial_\mu \phi^a \partial_\nu \phi^b$
- ϕ^a are four Stuckelberg fields.

Cosmology

- No flat and closed FRW solution for the model!

(G D'Amico, et al. PRD 2011)

- The Stuckelberg equation implies

$$m^2\partial_0(a^3 - a^2) = 0 \Rightarrow a = const.$$

- dRGT gravity has only the Minkowski solution as a flat FRW solution.

- But there is an open FRW solution (A Gumrukcuglu, et al. JCAP 2011)

$$3H^2 - \frac{3|K|}{a^2} = \rho_m + \Lambda_\pm,$$
$$-\frac{2\dot{H}}{N} - \frac{2|K|}{a^2} = \rho_m + p_m,$$

where

$$\Lambda_\pm \equiv -\frac{m^2}{(\alpha_3+\alpha_4)^2}\left[1+\alpha_3 \pm \sqrt{1+\alpha_3+\alpha_3^2-\alpha_4}\right] \times \left[1+\alpha_3^2-2\alpha_4 \pm (1+\alpha_3)\sqrt{1+\alpha_3+\alpha_3^2-\alpha_4}\right].$$

- linear perturbations on top of this solution shows that only 2 dof out of 5 dof of massive graviton are dynamical! (2 gravity wave polarizations) (A Gumrukcuglu, et al. JCAP 2012)
- 2 vector modes and one scalar mode lost at the linear level.
- Lost dof are associated with the maximally symmetric nature of FRW space-time.
- Going to axisymmetric Bianchi type I space-time of the form

$$ds^2 = -N^2 dt^2 + a^2 (e^{4\sigma} dx^2 + e^{-2\sigma} \delta_{ij} dy^i dy^j),$$

one can find all 5 dof at the linear level.

Cosmological perturbations

- At linear level the Lagrangian contains

$$\mathcal{L}^{(2)} \ni \dot{\mathcal{X}}_a \mathcal{K}_{ab} \dot{\mathcal{X}}_b, \qquad a, b = 1..5$$

  where $\mathcal{X}_a$ are gauge invariant dynamical variables and $\mathcal{K}_{ab}$ is the kinetic matrix. (A De Felis, et al. PRL 2012)
- However, one of these eigenvalues has a wrong sign, signaling instability.
- The self-accelerating solution becomes unstable; Among 5 dof of the massive graviton, one of them is ghost!
- 3 out of 5 eigenvalues of $\mathcal{K}_{ab}$ vanishes in the isotropy limit, i.e. $\sigma \to 0$.

- Massive gravity is a theory of spin-2 field with following free parameters in addition to the standard GR parameters:
  - Fiducial metric f_{ab}.
  - Graviton mass m.
  - Two dimensionless parameters $\alpha_{3,4}$.
- Making any of them dynamical can generalize the theory.
  - Making f_{ab} dynamical, leads to bi-metric and multi-metric gravities. (SF Hassan, et al. JHEP 2012; SS, et al. PRD 2012)
  - Making $m, \alpha_{3,4}$ dynamical leads to
    - Mass-varying theory (Q Huang, et al. PRD 2012)
    - Quasi-Dilaton (G D'Amico, et al. 2013)
    - Mimetic massive gravity (Z Haghani and SS, in preperation)
    - ...
- Let's continue with Quasi-Dilaton massive gravity.

Adding Quasi-Dilaton

- Add a scalar field which gives rise to an internal global symmetry.
- This new global symmetry is

$$x^\mu \to e^{\alpha} x^\mu \,, \quad g_{\mu\nu} \to e^{-2\alpha} g_{\mu\nu} \,,$$
$$\sigma \to \sigma - M_{\rm pl}\alpha \,, \quad \phi^a \to e^{\alpha}\phi^a \,.$$

- The scalar field is minimally coupled to the matter in the Einstein frame!
- The above, is a symmetry of the pure gravitational sector, not a symmetry of the full theory with matter.
- So, the Quasi-Dilaton!!

The Lagrangian in Einstein frame

- The Quasi-Dilaton Lagrangian in the Einstein frame as

(G D'Amico, et al. PRD 2013)

$$\mathcal{L} = \frac{M_{\rm Pl}^2}{2}\sqrt{-g}\Big[R - \frac{\omega}{M_{\rm pl}^2} g^{\mu\nu}\partial_\mu\sigma\partial_\nu\sigma - \frac{m^2}{4}\left(\mathcal{U}_2(\tilde{\mathcal{K}}) + \alpha_3\mathcal{U}_3(\tilde{\mathcal{K}}) + \alpha_4\mathcal{U}_4(\tilde{\mathcal{K}})\right)\Big] + \sqrt{-g}\mathcal{L}_m(g_{\mu\nu},\psi)\,,$$

where

$$\tilde{\mathcal{K}}^\mu{}_\nu = \delta^\mu_\nu - e^{\sigma/M_{\rm Pl}}\sqrt{g^{\mu\alpha}\partial_\alpha\phi^a\partial_\nu\phi^b\eta_{ab}}\,.$$

- The coupling of the scalar field σ to the matter is minimal because there is no Stuckelberg field in the matter sector.
- Derivative interactions to the matter is still allowed.

Flat FRW solution!

- Flat FRW solution exists in this theory (G D'Amico, et al. PRD 2013)

$$\left(3-\frac{\omega}{2}\right)M_{\rm Pl}^2H^2=\Lambda \quad \& \quad e^{\sigma(t)/M_{\rm Pl}}\propto a(t),$$

where

$$\Lambda = 3M_{\rm Pl}^2m^2\left[\frac{1}{4}(\alpha_3+4\alpha_4)c^3-(1+\frac{3}{2}\alpha_3+3\alpha_4)c^2 + (3+\frac{9}{4}\alpha_3+3\alpha_4)c-(2+\alpha_3+\alpha_4)\right],$$

and

$$c=\frac{3\alpha_3+8\alpha_4\pm\sqrt{9\alpha^3-64\alpha_4}}{8\alpha_4}$$

- Should meet the conditions

$$\boxed{0\le\omega<6} \quad \text{and} \quad 0<\alpha_4<\frac{\alpha_3^2}{8}.$$

Linear perturbations

- We have 2 healthy gravity wave modes (tensor sector), and 2 healthy vector modes with non-vanishing kinetic terms.

(SS, et al. PRD 2013)

- In the scalar sector, only 2 variables out of 5 scalar perturbations become dynamical, and can be written in a compact form
$$\mathcal{L} \ni \Phi'^T K_2 \Phi'$$
with
$$\Phi = \begin{pmatrix} \rho_1 \\ \rho_2 \end{pmatrix}.$$
and K_2 is a 2 dimensional kinetic matrix.

Scalar perturbations

- For the long wavelength limit, $k \to 0$, both eigenvalues of K_2 are positive, and non-zero. (SS, et al. PRD 2013)
- For the short wavelength limit, $k \gg m$, one can expand the eigenvalues of the matrix K_2 with the result

$$Z_1 \approx \frac{1}{2} a^2 \omega, \qquad Z_2 \approx \frac{a^6 (\omega - 6) \gamma_1^2}{48 \omega \boldsymbol{k}^4}$$

- With $\omega < 6$, the second eigenvalue is negative, signaling ghost instability in the scalar sector. (G D'Amico, et al. CQG 2013)
- Quasi-Dilaton solves the existence of flat FRW solution, and also the absence of 3 dof in linear level.
- However, one of the massive graviton dof is unstable on this background.

Extended quasi-dilaton

- One can add some quasi-dilaton self-interactions in a way that the ghost disappears from dof of massive graviton.
- We have added a scalar field, so the field configuration space will be 5 dimensional.
- So one should define the fiducial metric as (SS, et al. 2015)

$$f_{\mu\nu} = F_{AB}\partial_\mu\phi^A\partial_\nu\phi^B, \qquad \phi^A = (\phi^a, \sigma).$$

Extended quasi-dilaton

- The warped space-time can solve the problem

$$F_{AB}d\phi^A d\phi^B = e^{2\sigma/M_{Pl}}\eta_{ab}d\phi^a d\phi^b - \frac{\alpha_\sigma}{M_{Pl}^2 m^2}d\sigma^2,$$

- $\alpha_\sigma = 0$ corresponds to quasi-dilaton theory.
- Healthy condition is (A De Felis, et al. PLB 2014)

$$0 < \omega < 6, \qquad \text{and} \qquad X^2 < \frac{\alpha_\sigma H^2}{m^2} < r^2 X^2,$$

with

$$X = 1 + \frac{3\alpha_3 \pm \sqrt{9\alpha_3^2 - 12\alpha_4}}{2\alpha_4}, \qquad r = 1 + \frac{\omega H^2}{m^2 X^2\left[\alpha_3(X-1) - 2\right]}.$$

- For $\alpha_\sigma = 0$ the above condition can't be satisfied and the ghost returns.

Conclusions

- Massive gravity gives more dof to the graviton itself.
- The linear theory has vDVZ discontinuity problem, which can be cured by adding some non-linear interactions terms.
- In general non-linear interactions gives rise to BD ghost.
- dRGT is the only generaly covariant theory without vDVZ and BD!
- Massive graviton behaves ghosty on top of self-accelerating background.
- Coupling dRGT to a scalar field can solve the problem!

Thanks for your attention!