Can Solitons Support Thick Branes?

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Brane World

- The standard model and general relativity describe our universe very well, by considering it as four dimensional space time. However; they contain shortcomings such as hierarchy problem.

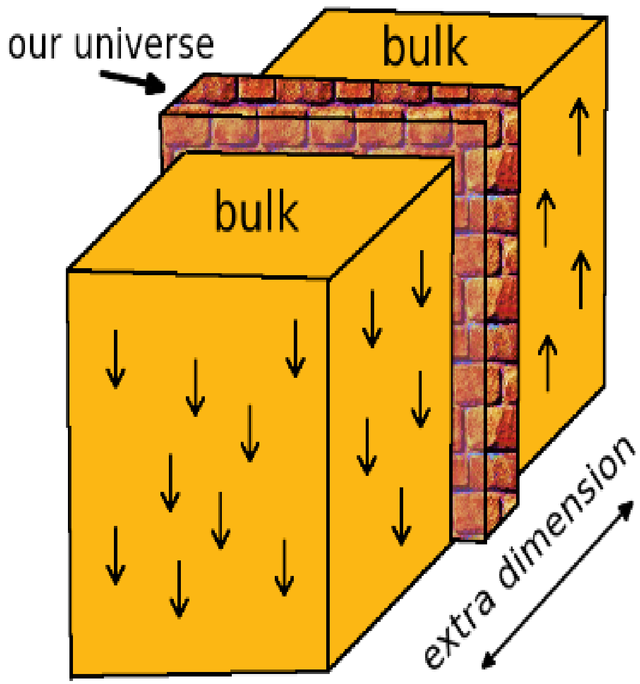
- One can imagine that our four-dimensional world as a brane-world or a surface layer in a five-dimensional bulk.

- **Domain-wall brane model building, From chirality to cosmology, Damien George,(2008)**

Domain Walls as a Brane

- Generally, the necessary condition of almost all extra-dimensional models is one or more background scalar fields.

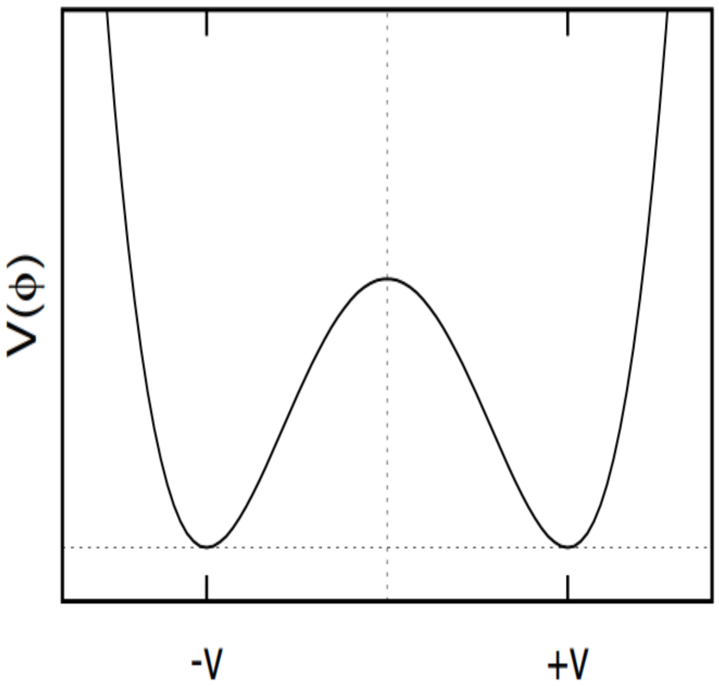
- Domain Walls forms as solution of scalar field (a kink).

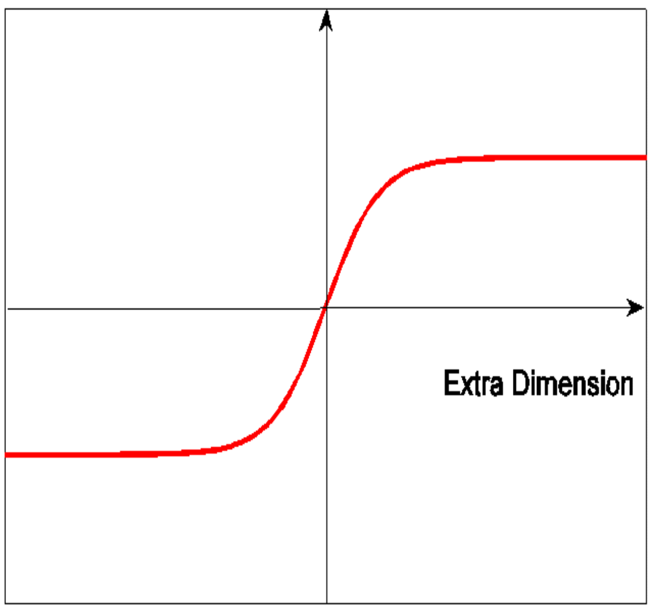


Domain-wall brane model building, From chirality to cosmology, Damien George,(2008)

Soliton models for the brane

- A domain-wall is a (thin) region separating two vacua.
- The field φ interpolates between two vacua as one moves along the extra dimension.





Thick Brane Formalism

- It would be quite valuable to investigate new models of the barne worlds which contain various kinds of poten

$$S=\int d^5x\sqrt{|g^{(5)}|}\left[\frac{1}{4}R[g^{(5)}]-\frac{1}{2}\partial_A\varphi\partial^A\varphi-V(\varphi)\right],$$

- The line element of the five-dimensional space-time can be written:

$$\begin{aligned} ds_5^2 &= g_{AB}dx^A dx^B \\ &= dw^2+e^{2A}(dx^2+dy^2+dz^2-dt^2), \end{aligned}$$

- 5-dimensional energy-momentum tensor is given by:

$$T_{AB} = \partial_A\varphi\partial_B\varphi - g_{AB}\left[\frac{1}{2}\partial_C\varphi\partial^C\varphi + V(\varphi)\right],$$

- Whenever gμν and φ depend only on w, the 5-dimensional gravitational and scalar field equations take the form:

$$3A'' + 6A'^2 = -k_5^2 e^{-2A} T_{00} = -k_5^2\left[\frac{1}{2}\varphi'^2 + V(\varphi)\right],$$

$$6A'^2 = k_5^2 T_{44} = k_5^2\left[\frac{1}{2}\varphi'^2 - V(\varphi)\right],$$

$$\varphi'' + 4A'\varphi' = \frac{dV(\varphi)}{d\varphi},$$

where the prime denotes a derivatives with respect to w.

- In order to obtain a first-order equation, we introduce an auxiliary function W according to some references, which demands:

$$A' = -\frac{1}{3}W(\varphi),$$
$$\varphi' = \frac{1}{2}\frac{\partial W(\varphi)}{\partial \varphi},$$

While V (φ) takes the following form :

$$V(\varphi) = \frac{1}{8}\left(\frac{\partial W(\varphi)}{\partial \varphi}\right)^2 - \frac{1}{3}W(\varphi)^2.$$

- Moreover, it is also interesting to calculate the geodesic equation along the fifth dimension in a thick brane, in order to investigate the particle motion near the brane. To this end, we start with the geodesic equation:

$$\frac{d^2x^0}{d\tau^2} + \Gamma^0_{AB}\frac{dx^A}{d\tau}\frac{dx^B}{d\tau} = 0 \quad \Rightarrow \quad \frac{d}{d\tau}\left(-2e^{2A}\dot{t}\right) = 0,$$

$$\frac{d^2x^4}{d\tau^2} + \Gamma^4_{AB}\frac{dx^A}{d\tau}\frac{dx^B}{d\tau} = 0 \quad \Rightarrow \quad \ddot{w} + A'e^{2A}\dot{t}^2 = 0,$$

Which eventually leads to :

$$\ddot{w} + F(w) = 0.$$ Where $$F(w) = -c_1^2 A'(w) e^{-2A(w)},$$

Sine-Gordon-based model

- The self-interaction potential of Sine-Gordon (SG) is:

$$\tilde{V}(\varphi) = \frac{a}{b}\left[1 - cos(b\varphi)\right] \, ,$$

- here a and b are free parameters of the model. The SG system has the following exact static kink solution :

$$\varphi(w) = \frac{4}{b}\arctan\left(e^{\sqrt{ab}w}\right),$$

- The corresponding potential for this model to support the is given by:

$$V(\varphi) = \frac{2a}{b}\sin^2\left(\frac{b\varphi}{2}\right) - \frac{64a}{3b^3}\left[1+\cos\left(\frac{b\varphi}{2}\right)\right]^2 ,$$

- Notice that this potential has two series nondegeneratevacuua, as in the DSG (double sine-G system potential. However, in the limit of b≫a these vacuu to the same value (become degenerate).
- In the limits of $w \to \pm\infty$, the Ricci scalar becomes:

$$\lim_{w\to+\infty} R = 0,$$

$$\lim_{w\to-\infty} R = -\frac{5120}{9}\frac{a}{b^3},$$

$$\lim_{w\to 0} R = \frac{64}{9}\frac{(-20+3b^2)a}{b^3}.$$

$$\lim_{w\to +\infty} G^A_B \; = \; 0,$$

$$\lim_{w\to -\infty} G^A_B \; = \; \frac{512}{3}\frac{a}{b^3},$$

$$\lim_{w\to 0} G^\mu_\nu = \frac{8}{3}\frac{(16-3b^2)a}{b^3}\,\delta^\mu_\nu\,,$$

The geodesic equation

- The confining gravitational field of the brane is best obser
looking at the geodesic equation of a test particle moving
the direction of the extra dimension, which is given by:

$$\ddot{w} + c_1^2 \frac{8}{9}\frac{32^{\frac{1}{b^2}}\, 2^{\frac{1}{3b^2}}\, a\left(3b^2+16\right)}{b^3}\, w \approx c_1^2 \frac{8}{3}\frac{32^{\frac{1}{b^2}}\, 2^{\frac{1}{3b^2}}\sqrt{ab}}{b^2}.$$

- If [illegible] n assign an e
to each quantum state given by $E_n = (n + 1/2)\hbar\omega$,where:

$$\hbar\omega = \Omega = \sqrt{F'(w_0)} \approx c_1 \sqrt{\frac{8}{9}\frac{32^{\frac{1}{b^2}}\, 2^{\frac{1}{3b^2}}\, a(3b^2+16)}{b^3}}.$$

φ^4-based model

- For the φ^4-based model, we have :

$$\tilde{V}(\varphi) = \frac{\beta^2}{2\alpha^2}\left(\varphi^2 - \alpha^2\right)^2,$$

- Where α and β are solution reads:

$$\varphi(w) = \alpha \tanh(\beta w),$$

- The modified potential is obtained as:

$$V(\varphi) = \frac{1}{2}\alpha^2\beta^2\left(1 - \frac{\varphi^2}{\alpha^2}\right)^2 - \frac{4}{27}\varphi^2\alpha^2\beta^2\left(3 - \frac{\varphi^2}{\alpha^2}\right)^2 ,$$

$$\lim_{w\to+\infty} G_B^A = \frac{32}{27}\alpha^4\beta^2,$$
$$\lim_{w\to-\infty} G_B^A = \frac{32}{27}\alpha^4\beta^2,$$

$$\lim_{w\to 0} G_\nu^\mu = -2\alpha^2\beta^2\delta_\nu^\mu .$$

The geodesic equation

- The geodesic equation for a test particle moving in the direction of the fth dimension one obtains:

$$\ddot{w} + c_1^2 \frac{2}{3} \alpha^2 \beta^2 w = 0 \,,$$

- which corresponds to a linearized quantum mode of energy:

$$\hbar\omega = \Omega = \sqrt{F'(w_0)} = \sqrt{\tfrac{2}{3}} c_1 \alpha \beta .$$

φ^6-based model

- For the φ^6-based model, we have :

- and as a result, the $\tilde{V}(\varphi) = \frac{\beta^2}{4\alpha^2}\varphi^2\left(\varphi^2 - \alpha^2\right)^2$, y :

- α and β are constant as well

$$\phi(w) = \frac{\alpha}{\sqrt{1 + e^{(-\sqrt{2}\alpha\beta w)}}},$$

- Thus, for the φ^6 system the self-interaction potenti
form:

$$V(\varphi) = \frac{1}{4}\frac{\beta^2\varphi^2\left(\alpha^2-\varphi^2\right)^2}{\alpha^2} - \frac{1}{24}\frac{\beta^2\varphi^4\left(2\alpha^2-\varphi^2\right)^2}{\alpha^2},$$

$$\lim_{w\to+\infty} G^A_B = \frac{1}{12}\alpha^6\beta^2,$$

$$\lim_{w\to-\infty} G^A_B = 0,$$

$$\lim_{w\to 0} G^\mu_\nu = \frac{1}{8}\alpha^4\beta^2\left(\frac{3}{8}\alpha^2-1\right)\delta^\mu_\nu\,.$$

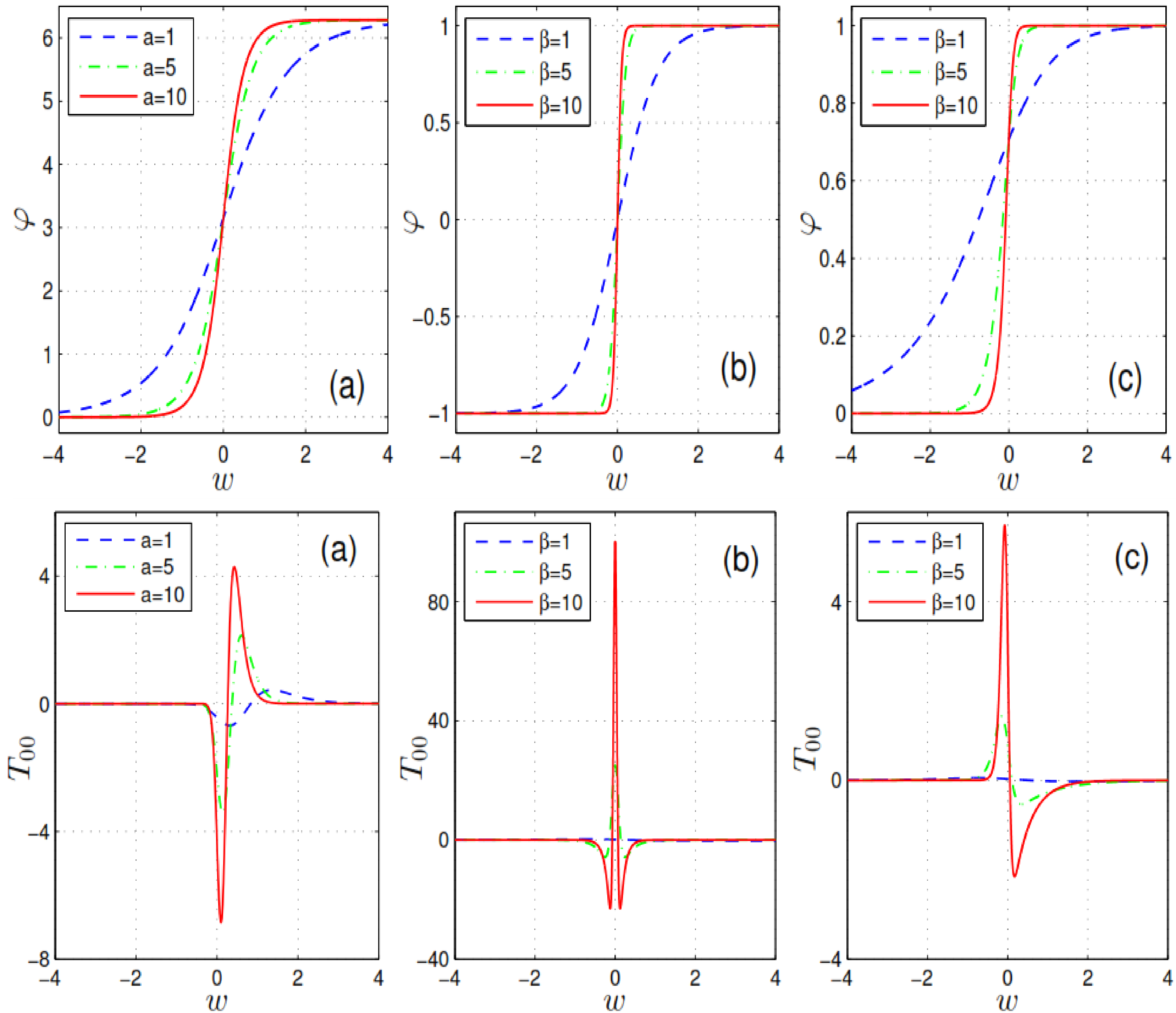
The geodesic equation

- which leads to the following geodesic equation:

$$\ddot{w} + c_1^2 \frac{1}{192} \exp\left[\frac{\alpha^2}{12}\left(2\ln 2 + 1\right)\right] \alpha^4 \beta^2 \left(8 + 3\alpha^2\right) w \approx \frac{1}{16} \exp\left[\frac{1}{12}\alpha^2\left(2\ln 2 + 1\right)\right] \alpha^3 \beta \sqrt{2}\,.$$

- The quantum mode energy is thus given by:

$$\hbar\omega = \Omega = \sqrt{F'(w_0)} \approx c_1 \frac{\alpha^2 \beta}{24} \sqrt{3} \sqrt{\exp\left(\frac{\alpha^2}{12}\left(2\ln(2) + 1\right)\right)\left(8 + 3\alpha^2\right)}\,.$$



a=1
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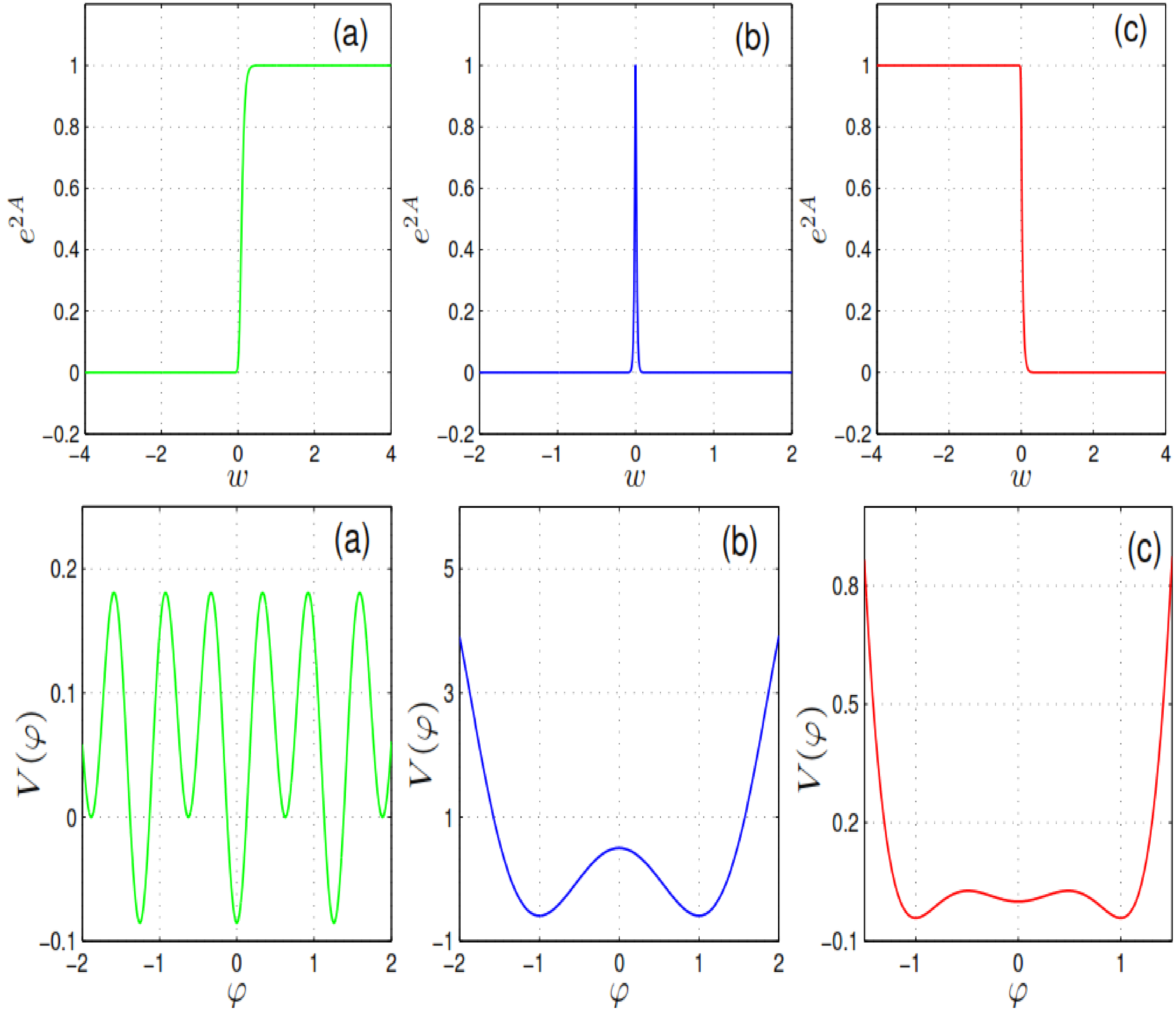
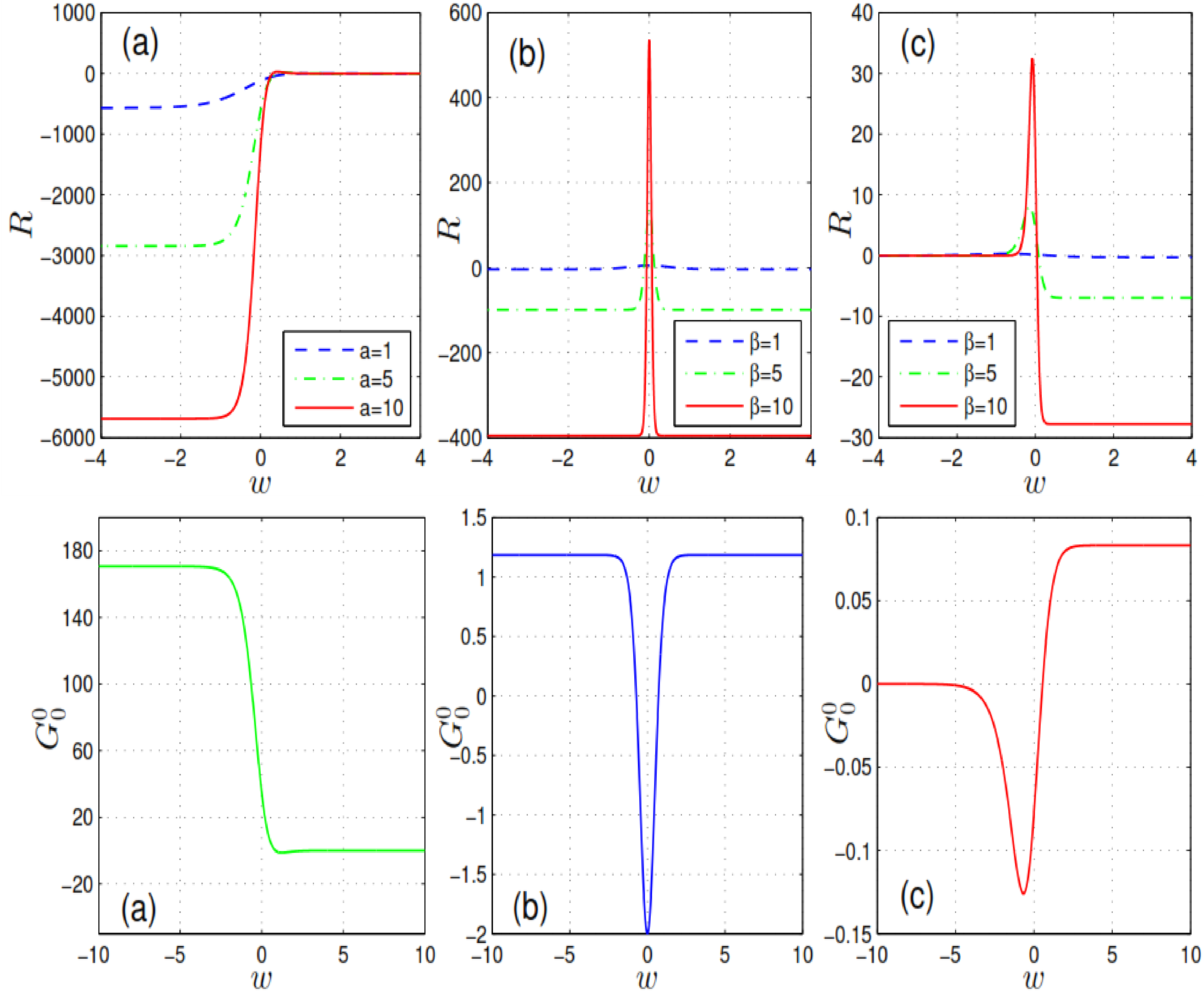


(a)
(b)
(c)
e^{2A}
w
$V(\varphi)$
φ


(a)
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Brane stability

- We choose an "axial gauge" where the metric is perturbed as, study the stability of the branes:

$$ds^2 = e^{2A(w)}(g_{\mu\nu} + \varepsilon h_{\mu\nu})dx^\mu dx^\nu - dw^2,$$

- Where ε a r and the metric perturbations respectively.
- The corresponding Schrodinger equation takes the form:

$$-\frac{d^2\psi(z)}{dz^2} + U(z)\psi(z) = k^2\psi(z),$$

- where the potential is given by:

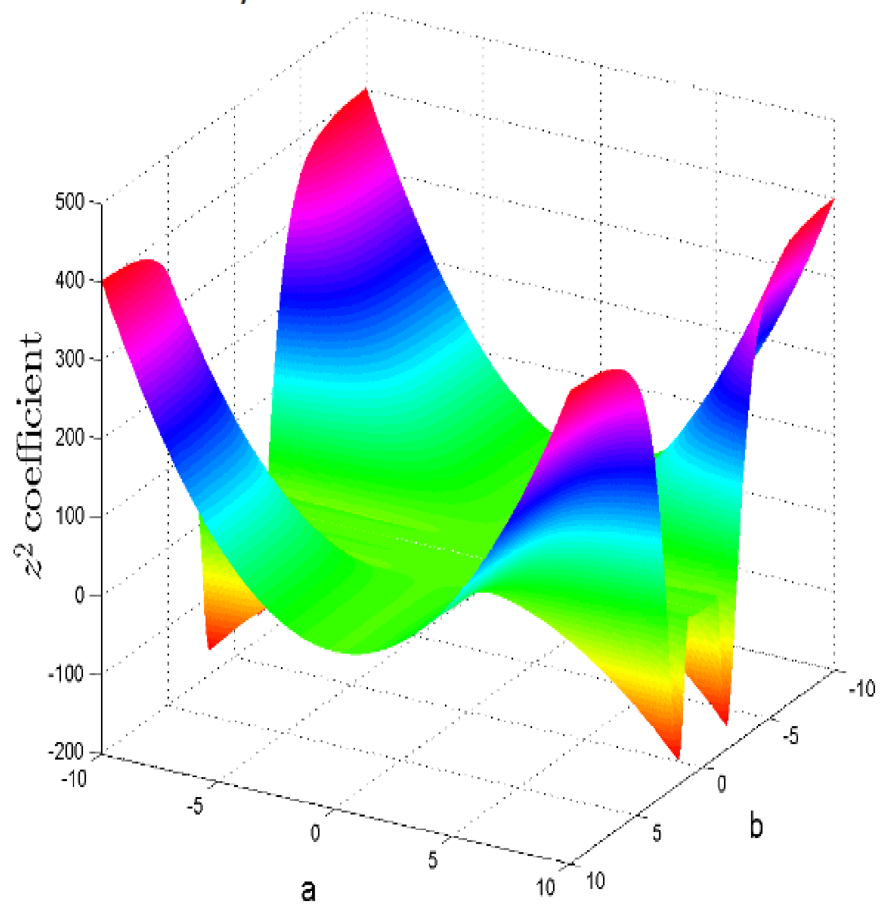
$$U(z) = -\frac{9}{4}\Lambda + \frac{9}{4}A'^2 + \frac{3}{2}A''.$$

- Note that is a cosmological constant on the brane, which c
be positive, negative or zero corresponding to the 4D space
being de Sitter (dS_4), anti-de Sitter (AdS_4)or Minkowski
respectively.

- The corresponding potentials for the SG, φ^4 and φ^6 system
calculated to be:

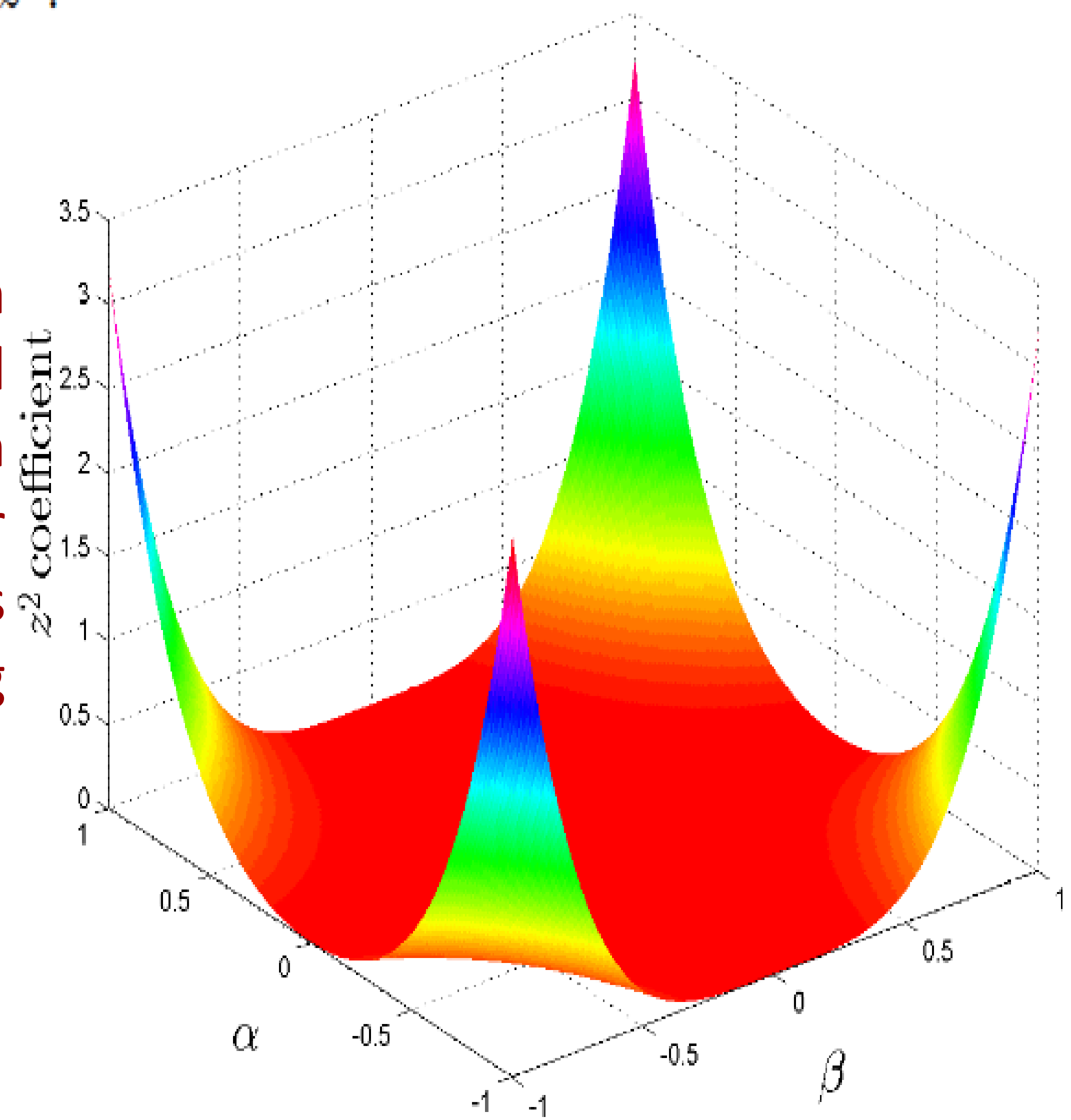
$$
\begin{aligned}
U(z) \;=\; & -\frac{1}{162b^9}a\left(-2592b^6 16^{\frac{-1}{b^2}} 2^{\frac{-4}{3b^2}} - 1728\times 32^{\frac{-1}{b^2}} 2^{\frac{-1}{3b^2}} b^6 + 648\times 32^{\frac{-1}{b^2}} 2^{\frac{-1}{3b^2}} b^8\right) \\
& -\frac{1}{162b^9}a\left(-36256^{\frac{b^2-1}{b^2}}\sqrt{a}bb^4 + 27256^{\frac{b^2-1}{b^2}}\sqrt{a}bb^6 + 5184\sqrt{a}b4^{\frac{-1}{b^2}} 2^{\frac{-1}{b^2}} b^6 32^{\frac{-1}{b^2}} - 13824\sqrt{a}b4^{\frac{-1}{b^2}} 2^{\frac{-1}{b^2}} b^4 32^{\frac{-1}{b^2}}\right) z \\
& -\frac{1}{162b^9}a\left(-55296a1024^{\frac{-1}{b^2}} 2^{\frac{-2}{3b^2}} b^3 - 36864ab^3 4^{\frac{-1}{b^2}} 2^{\frac{-2}{3b^2}} 256^{\frac{-1}{b^2}} - 6048a1024^{\frac{-1}{b^2}} 2^{\frac{-2}{3b^2}} b^7\right. \\
& \left. +27648ab^5 4^{\frac{-1}{b^2}} 2^{\frac{-2}{3b^2}} 256^{\frac{-1}{b^2}} + 55296a1024^{\frac{-1}{b^2}} 2^{\frac{-2}{3b^2}} b^5 - 648a1024^{\frac{-1}{b^2}} 2^{\frac{-2}{3b^2}} b^9\right) z^2.
\end{aligned}
$$

The coefficient of the z^2 term in the potential of the linearized Schrodinger equation as a function of the free parameters a and b for the SG system. Negative values correspond to a first order instability.



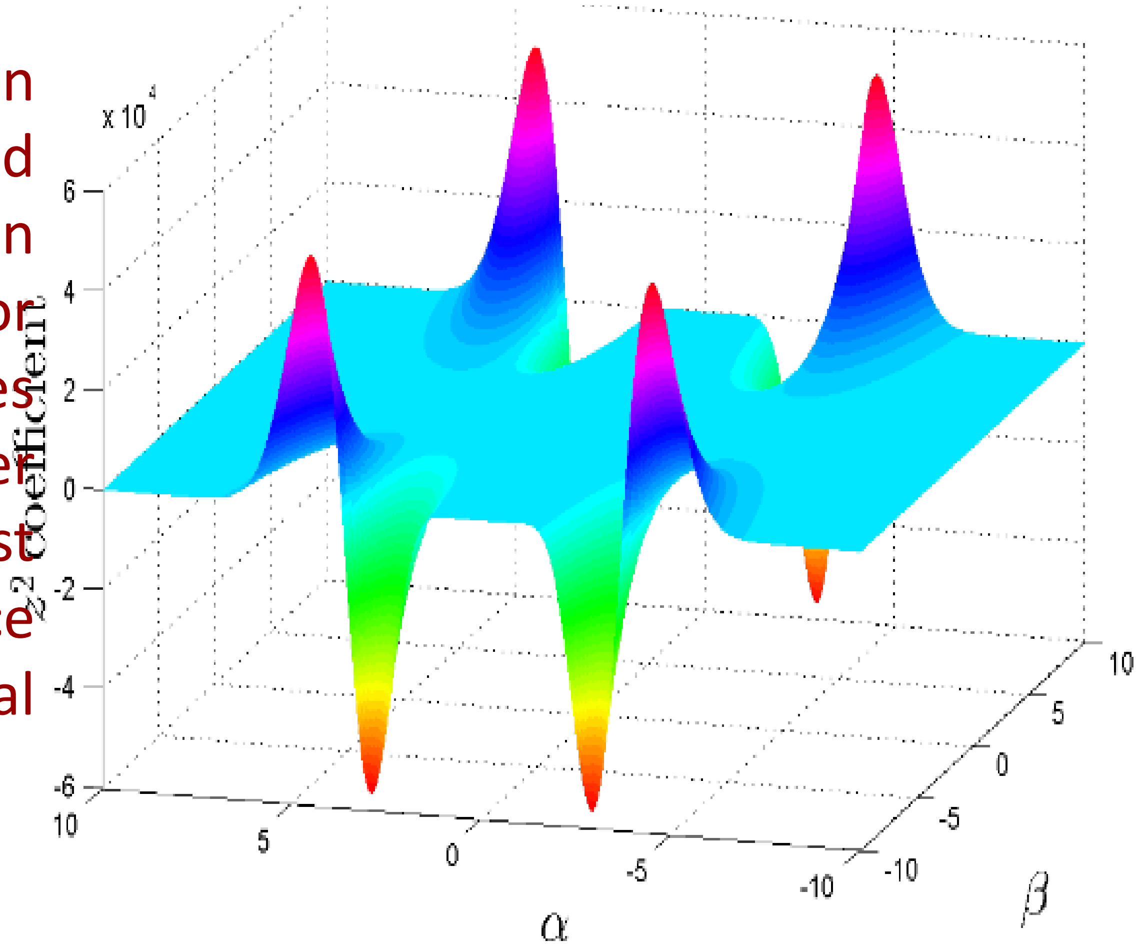
$$U(z) = \frac{3}{2} - \frac{2}{3}\alpha^2\beta^2 + \left(\frac{17}{9}\alpha^4\beta^4 + \frac{4}{3}\alpha^2\beta^4\right) z^2.$$

The coefficient of the z^2 term in the potential of the linearized Schrodinger equation as a function of the free parameters α and β for the φ^4 system. The coefficient is everywhere positive, signaling linear stability.



$$U(z) = \frac{1}{512}\alpha^4\beta^2\exp\left[-\frac{1}{12}\alpha^2(1+2\ln 2)\right](-32+15\alpha^2)$$
$$-\frac{1}{4096}\alpha^5\beta^3\sqrt{2}\exp\left[-\frac{1}{8}\alpha^2(1+2\ln 2)\right](-112\alpha^2+15\alpha^4-128)\,z$$
$$+\frac{1}{196608}\alpha^6\beta^4\exp\left[-\frac{1}{6}\alpha^2(1+2\ln 2)\right](-1728\alpha^4+6144+135\alpha^6-2048\alpha^2)\,z^2.$$

The coefficient of the z^2 term in the potential of the linearized Schrodinger equation as a function of the free parameters α and β for the φ^6 system. Negative values correspond to a first order instability. There are also vast patches in the parameter space which have almost neutral stability.



Conclusion

The confining effect of the scalar field in all these three models were confirmed by examining the geodesic equation for a test particle moving normal to the brane.

In particular, it turns out that the modified potential fo[r] system resembles that of the double sine-Gordon (DSG) while those of φ^4 and φ^6 became φ^6 and φ^8 , respectively.

We considered the limiting case in which the brane tends to zero thickness and approaches a thin brane. It should be noted that the topological stability of the soliton brane remains valid even in this limit (at least at the classical level).

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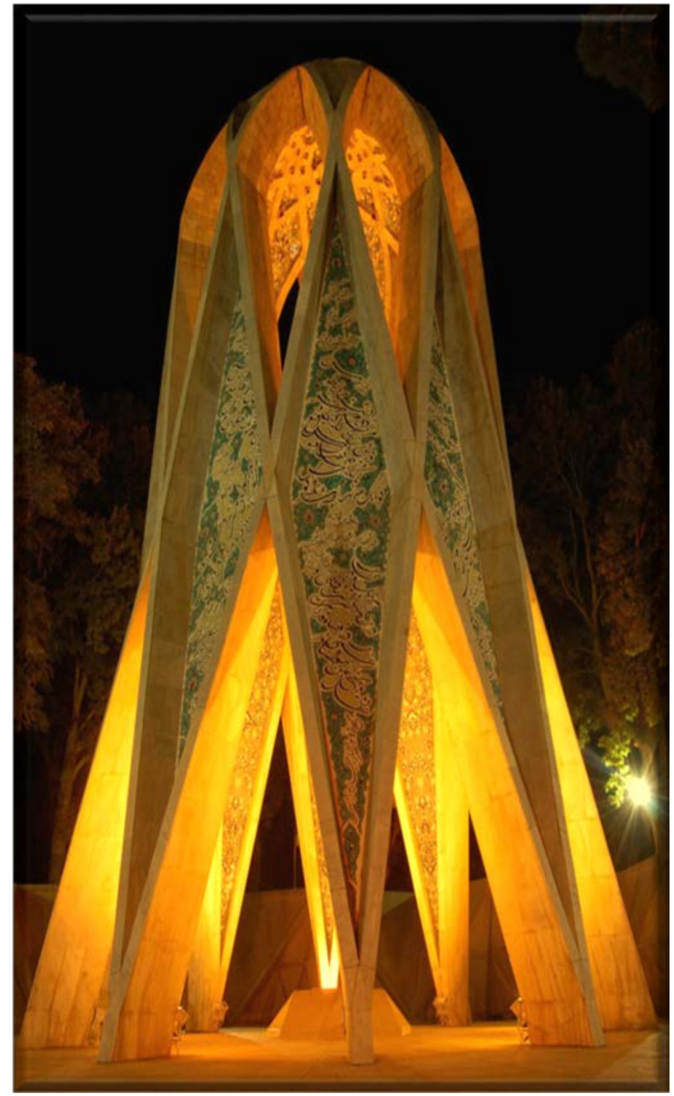
هرگز دل من ز علم محروم نشد
کم ماند ز اسرار که معلوم نشد
هفتاد و دو سال فکر کردم شب و روز
معلومم شد که هیچ معلوم نشد

Never my heart deprived of knowledge

Few secrets were not divulged

Foe seventy two years I pondered day and night

Only to know that I know nothing



Tomb of Omar Khayyám Neishapuri in Nishapur, Iran

Thank You

Omar Khayyám (/ˈoʊmɑr kaɪˈjɑːm, -ˈjæm, ˈoʊmər/; Persian: غیاث‌الدین ابوالفتح عمر ابراهیم خیام نیشابوری, pronounced [xæjˈjɒːm]; 18 May 1048 – 4 December 1131), was a Persian mathematician, astronomer, philosopher, and poet. He also wrote treatises on mechanics, geography, mineralogy and music.