Flat space holography

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1402.3687, 1307.4768, 1305.2919, 1208.4372, 1208.1658
Statement of the main result

Hot flat space

$ds^2 = \pm dt^2 + dr^2 + r^2 d\varphi^2$

$(\varphi \sim \varphi + 2\pi)$
Statement of the main result

Hot flat space

\[ ds^2 = \pm dt^2 + dr^2 + r^2 \, d\varphi^2 \]

\[ (\varphi \sim \varphi + 2\pi) \]

Flat space cosmology

\[ ds^2 = \pm d\tau^2 + \frac{(E\tau)^2 \, dx^2}{1 + (E\tau)^2} + (1 + (E\tau)^2) \left( dy + \frac{(E\tau)^2}{1 + (E\tau)^2} \, dx \right)^2 \]

\[ (y \sim y + 2\pi r_0) \]

Bagchi, Detournay, Grumiller & Simon '13
Outline

Motivation: Gravity in lower dimensions

Review: AdS/CFT from a relativist’s perspective

Developments: Flat space holography

Novel result: Cosmic phase transition
Outline

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Motivation for studying gravity in 2 and 3 dimensions

- **Quantum gravity**
  - Address conceptual issues of quantum gravity
  - Black hole evaporation, information loss, black hole microstate counting, virtual black hole production, ... 
  - Technically much simpler than 4D or higher D gravity
  - Integrable models: powerful tools in physics
  - Models should be as simple as possible, but not simpler

- **Gauge/gravity duality + indirect physics applications**
  - Deeper understanding of black hole holography
  - $\text{AdS}_3/\text{CFT}_2$ correspondence best understood
  - Quantum gravity via AdS/CFT
  - Applications to 2D condensed matter systems
  - Gauge gravity duality beyond standard AdS/CFT: warped AdS, Lifshitz, Schrödinger, non-relativistic or log CFTs, higher spin holography ...

- **Flat space holography**

- **Direct physics applications**
  - Cosmic strings
  - Black hole analog systems in condensed matter physics
  - Effective theory for gravity at large distances
Outline

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Novel result: Cosmic phase transition
Holographic algorithm from gravity point of view

Universal recipe:

1. Identify bulk theory and variational principle
   Example: Einstein gravity with Dirichlet boundary conditions
   \[ I = -\frac{1}{16\pi G_N} \int d^3 x \sqrt{|g|} \left( R + \frac{2}{\ell^2} \right) \]
   with \( \delta g = \) fixed at the boundary
Holographic algorithm from gravity point of view

Universal recipe:

1. Identify bulk theory and variational principle
2. Fix background and impose suitable boundary conditions
   Example: asymptotically AdS
   \[ ds^2 = d\rho^2 + \left( e^{2\rho/\ell} \gamma_{ij}^{(0)} + \gamma_{ij}^{(2)} + \ldots \right) \, dx^i \, dx^j \]
   with \( \delta \gamma^{(0)} = 0 \) for \( \rho \to \infty \)
Holographic algorithm from gravity point of view

Universal recipe:

1. Identify bulk theory and variational principle
2. Fix background and impose suitable boundary conditions
3. Perform canonical analysis and check consistency of bc’s
   Example: Brown–Henneaux analysis for 3D Einstein gravity

\[ \{ Q[\varepsilon], Q[\eta] \} = \delta_\varepsilon Q[\eta] \]

with

\[ Q[\varepsilon] \sim \int d\varphi \, L(\varphi) \varepsilon(\varphi) \]

and

\[ \delta_\varepsilon L = -L \varepsilon - 2L \varepsilon' - \frac{l}{16\pi G_N} \varepsilon''' \]

4. Derive (classical) asymptotic symmetry algebra and central charges
5. Improve to quantum ASA
6. Study unitary representations of quantum ASA
7. Identify/constrain dual field theory
8. If unhappy with result go back to previous items and modify

Apply algorithm above to flat space holography in 3D gravity

Goal of this talk:

Daniel Grumiller — Flat space holography
Holographic algorithm from gravity point of view

Universal recipe:
1. Identify bulk theory and variational principle
2. Fix background and impose suitable boundary conditions
3. Perform canonical analysis and check consistency of bc’s
4. Derive (classical) asymptotic symmetry algebra and central charges
   Example: Two copies of Virasoro algebra
   \[
   [\mathcal{L}_n, \mathcal{L}_m] = (n - m) \mathcal{L}_{n+m} + \frac{c}{12} (n^3 - n) \delta_{n+m,0}
   \]
   with Brown–Henneaux central charge
   \[
   c = \frac{3\ell}{2G_N}
   \]
   Reminder: ASA = quotient algebra of asymptotic symmetries by ‘trivial’ asymptotic symmetries with zero canonical charges
Holographic algorithm from gravity point of view

Universal recipe:
1. Identify bulk theory and variational principle
2. Fix background and impose suitable boundary conditions
3. Perform canonical analysis and check consistency of bc’s
4. Derive (classical) asymptotic symmetry algebra and central charges
5. Improve to quantum ASA
   Example: semi-classical ASA in spin-3 gravity (Henneaux, Rey ’10; Campoleoni, Pfenninger, Fredenhagen, Theisen ’10)

\[
[W_n, W_m] = \frac{16}{5c} \sum_p L_p L_{n+m-p} + \ldots
\]

quantum ASA

\[
[W_n, W_m] = \frac{16}{5c + 22} \sum_p : L_p L_{n+m-p} : + \ldots
\]
Holographic algorithm from gravity point of view

Universal recipe:

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2. Fix background and impose suitable boundary conditions
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5. Improve to quantum ASA
6. Study unitary representations of quantum ASA

Example:

Afshar et al ’12
Discrete set of Newton constant values compatible with unitarity
(3D spin-N gravity in next-to-principal embedding)
Holographic algorithm from gravity point of view

Universal recipe:
1. Identify bulk theory and variational principle
2. Fix background and impose suitable boundary conditions
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4. Derive (classical) asymptotic symmetry algebra and central charges
5. Improve to quantum ASA
6. Study unitary representations of quantum ASA
7. Identify/constrain dual field theory
   Example: Monster CFT in (flat space) chiral gravity
   Witten ’07
   Li, Song & Strominger ’08
   Bagchi, Detournay & Grumiller ’12

\[ Z(q) = J(q) = \frac{1}{q} + (1 + 196883)q + O(q^2) \]

Note: \( \ln 196883 \approx 12.2 = 4\pi \) + quantum corrections
Holographic algorithm from gravity point of view

Universal recipe:
1. Identify bulk theory and variational principle
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Examples: too many!
Holographic algorithm from gravity point of view

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Apply algorithm above to flat space holography in 3D gravity
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Motivation: Gravity in lower dimensions

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Developments: Flat space holography

Novel result: Cosmic phase transition
Flat space from contraction of AdS

Idea: take $\ell \to \infty$ limit of AdS results!
Flat space from contraction of AdS

Idea: take $\ell \to \infty$ limit of AdS results!

- Works straightforwardly sometimes, otherwise not

Example where it works nicely: asymptotic symmetry algebra

Take linear combinations of Virasoro generators $L_n, \bar{L}_n$

$[L_n, L_m] = (n - m) L_{n+m}$

$[L_n, M_m] = (n - m) M_{n+m} + c M_{1/2} (n^3 - n) \delta_{n+m,0}$

$[M_n, M_m] = 0$

This is nothing but the BMS$_3$ algebra (or GCA$_2$)!

Ashtekar, Bicak & Schmidt '96

Example where it does not work easily: boundary conditions!
Flat space from contraction of AdS

**Idea:** take $\ell \to \infty$ limit of AdS results!

- Works straightforwardly sometimes, otherwise not
- Example where it works nicely: asymptotic symmetry algebra

\[
\mathcal{L}_n = \mathcal{L}_n - \bar{\mathcal{L}}_n = \frac{1}{\ell} (\mathcal{L}_n + \bar{\mathcal{L}}_n)
\]

Make \textit{In}\textit{"o}n\textit{"u–Wigner contraction $\ell \to \infty$ on ASA}

\[
\begin{align*}
[\mathcal{L}_n, \mathcal{L}_m] &= (n - m) \mathcal{L}_n + m + c \mathcal{L}_{12} (n^3 - n) \delta_{n+m,0} \\
[\mathcal{L}_n, \mathcal{M}_m] &= (n - m) \mathcal{M}_n + m + c \mathcal{M}_{12} (n^3 - n) \delta_{n+m,0} \\
[\mathcal{M}_n, \mathcal{M}_m] &= 0
\end{align*}
\]

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- Example where it does not work easily: boundary conditions!
Flat space from contraction of AdS

Idea: take $\ell \to \infty$ limit of AdS results!

- Works straightforwardly sometimes, otherwise not
- Example where it works nicely: asymptotic symmetry algebra
- Take linear combinations of Virasoro generators $\mathcal{L}_n, \bar{\mathcal{L}}_n$

$$L_n = \mathcal{L}_n - \bar{\mathcal{L}}_{-n} \quad M_n = \frac{1}{\ell} (\mathcal{L}_n + \bar{\mathcal{L}}_{-n})$$

This is nothing but the BMS$_3$ algebra (or GCA$_2$)!

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$$L_n = L_n - \bar{L}_{-n} \quad M_n = \frac{1}{\ell} (L_n + \bar{L}_{-n})$$

- Make Inönü–Wigner contraction $\ell \to \infty$ on ASA

$$[L_n, L_m] = (n - m) L_{n+m} + \frac{c_L}{12} (n^3 - n) \delta_{n+m,0}$$

$$[L_n, M_m] = (n - m) M_{n+m} + \frac{c_M}{12} (n^3 - n) \delta_{n+m,0}$$

$$[M_n, M_m] = 0$$

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- Make Inönü–Wigner contraction $\ell \to \infty$ on ASA

$$[L_n, L_m] = (n - m) L_{n+m} + \frac{c_L}{12} (n^3 - n) \delta_{n+m,0}$$
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Flat space from contraction of AdS

Idea: take $\ell \to \infty$ limit of AdS results!

- Works straightforwardly sometimes, otherwise not
- Example where it works nicely: asymptotic symmetry algebra
- Take linear combinations of Virasoro generators $L_n$, $\bar{L}_n$
  \[
  L_n = L_n - \bar{L}_{-n} \quad M_n = \frac{1}{\ell} \left( L_n + \bar{L}_{-n} \right)
  \]
- Make Inönü–Wigner contraction $\ell \to \infty$ on ASA
  \[
  [L_n, L_m] = (n - m) L_{n+m} + \frac{c_L}{12} (n^3 - n) \delta_{n+m,0}
  \]
  \[
  [L_n, M_m] = (n - m) M_{n+m} + \frac{c_M}{12} (n^3 - n) \delta_{n+m,0}
  \]
  \[
  [M_n, M_m] = 0
  \]
- This is nothing but the BMS$_3$ algebra (or GCA$_2$)!
  Ashtekar, Bicak & Schmidt '96
- Example where it does not work easily: boundary conditions!
Apply algorithm just described

1. Identify bulk theory and variational principle
   Topologically massive gravity with mixed boundary conditions
   \[ I = I_{EH} + \frac{1}{32\pi G} \int d^3x \sqrt{-g} \varepsilon^{\lambda\mu\nu} \Gamma^\rho_{\lambda\sigma} \left( \partial_\mu \Gamma^\sigma_{\nu\rho} + \frac{2}{3} \Gamma^\sigma_{\mu\tau} \Gamma^\tau_{\nu\rho} \right) \]
   with \( \delta g = \text{fixed} \) and \( \delta K_L = \text{fixed} \) at the boundary

Deser, Jackiw & Templeton '82
Apply algorithm just described

1. Identify bulk theory and variational principle
2. Fix background and impose suitable boundary conditions asymptotically flat adapted to lightlike infinity \((\varphi \sim \varphi + 2\pi)\)

\[
\text{d}\tilde{s}^2 = - \text{d}u^2 - 2 \text{d}u \text{d}r + r^2 \text{d}\varphi^2
\]

\[
\begin{align*}
g_{uu} &= h_{uu} + O(\frac{1}{r}) \\
g_{ur} &= -1 + h_{ur}/r + O(\frac{1}{r^2}) \\
g_{u\varphi} &= h_{u\varphi} + O(\frac{1}{r}) \\
g_{rr} &= h_{rr}/r^2 + O(\frac{1}{r^3}) \\
g_{r\varphi} &= h_1(\varphi) + h_{r\varphi}/r + O(\frac{1}{r^2}) \\
g_{\varphi\varphi} &= r^2 + (h_2(\varphi) + uh_3(\varphi))r + O(1)
\end{align*}
\]

Barnich & Compere '06
Bagchi, Detournay & Grumiller '12
Apply algorithm just described

1. Identify bulk theory and variational principle
2. Fix background and impose suitable boundary conditions
3. Perform canonical analysis and check consistency of bc’s

Obtain canonical boundary charges

\[ Q_{Mn} = \frac{1}{16\pi G} \int d\varphi e^{in\varphi} (h_{uu} + h_3) \]

\[ Q_{Ln} = \frac{1}{16\pi G\mu} \int d\varphi e^{in\varphi} (h_{uu} + \partial_u h_{ur} + \frac{1}{2} \partial_u^2 h_{rr} + h_3) \]

\[ + \frac{1}{16\pi G} \int d\varphi e^{in\varphi} (inuh_{uu} + inh_{ur} + 2h_{w\varphi} + \partial_u h_{r\varphi} \]

\[ - (n^2 + h_3)h_1 - inh_2 - in\partial_\varphi h_1) \]

Bagchi, Detournay & Grumiller ’12
Apply algorithm just described

1. Identify bulk theory and variational principle
2. Fix background and impose suitable boundary conditions
3. Perform canonical analysis and check consistency of bc’s
4. Derive (classical) asymptotic symmetry algebra and central charges

\[
[L_n, L_m] = (n - m) L_{n+m} + \frac{c_L}{12} (n^3 - n) \delta_{n+m,0}
\]

\[
[L_n, M_m] = (n - m) M_{n+m} + \frac{c_M}{12} (n^3 - n) \delta_{n+m,0}
\]

\[
[M_n, M_m] = 0
\]

with central charges

\[
c_L = \frac{3}{\mu G} \quad \quad c_M = \frac{3}{G}
\]

Note:

- \( c_L = 0 \) in Einstein gravity
- \( c_M = 0 \) in conformal Chern–Simons gravity (\( \mu \to 0, \mu G = \frac{1}{8k} \))

Flat space chiral gravity!

Bagchi, Detournay & Grumiller ’12
Apply algorithm just described

1. Identify bulk theory and variational principle
2. Fix background and impose suitable boundary conditions
3. Perform canonical analysis and check consistency of bc’s
4. Derive (classical) asymptotic symmetry algebra and central charges
5. Improve to quantum ASA
   Trivial here
Apply algorithm just described

1. Identify bulk theory and variational principle
2. Fix background and impose suitable boundary conditions
3. Perform canonical analysis and check consistency of bc’s
4. Derive (classical) asymptotic symmetry algebra and central charges
5. Improve to quantum ASA
6. Study unitary representations of quantum ASA
   - Straightforward in flat space chiral gravity
   - Difficult/impossible otherwise

What about non-perturbative states analogue to BTZ black holes?
Where/what are they in flat space (chiral) gravity?

But...:
Apply algorithm just described

1. Identify bulk theory and variational principle
2. Fix background and impose suitable boundary conditions
3. Perform canonical analysis and check consistency of bc’s
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   Monster CFT in flat space chiral gravity
   Witten ’07
   Li, Song & Strominger ’08
   Bagchi, Detournay & Grumiller ’12

\[ Z(q) = J(q) = \frac{1}{q} + (1 + 196883) q + O(q^2) \]

Note: \( \ln 196883 \approx 12.2 = 4\pi + \text{quantum corrections} \)
Apply algorithm just described

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We are
Apply algorithm just described

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But....:

What about non-perturbative states analogue to BTZ black holes? Where/what are they in flat space (chiral) gravity?
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Novel result: Cosmic phase transition
Flat space cosmologies (Cornalba & Costa ’02)

Start with BTZ in AdS:

\[
ds^2 = -\frac{(r^2 - R_+^2)(r^2 - r_-^2)}{r^2 \ell^2} \, dt^2 + \frac{r^2 \ell^2 \, dr^2}{(r^2 - R_+^2)(r^2 - r_-^2)} + r^2 \left( d\varphi - \frac{R_+ r_-}{\ell r^2} \, dt \right)^2
\]
Flat space cosmologies (Cornalba & Costa ’02)

- Start with BTZ in AdS:

\[ ds^2 = -\frac{(r^2 - R_+^2)(r^2 - r_-^2)}{r^2\ell^2} \, dt^2 + \frac{r^2\ell^2 \, dr^2}{(r^2 - R_+^2)(r^2 - r_-^2)} + r^2 \left( d\varphi - \frac{R_+ r_-}{\ell r^2} \, dt \right)^2 \]

- Consider region between the two horizons \( r_- < r < R_+ \)
Flat space cosmologies (Cornalba & Costa '02)

▶ Start with BTZ in AdS:
\[ ds^2 = - \frac{(r^2 - R_+^2)(r^2 - r_-^2)}{r^2 \ell^2} dt^2 + \frac{r^2 \ell^2 \, dr^2}{(r^2 - R_+^2)(r^2 - r_-^2)} + r^2 \left( d\varphi - \frac{R_+ r_-}{\ell r^2} \, dt \right)^2 \]

▶ Consider region between the two horizons \( r_- < r < R_+ \)

▶ Take the \( \ell \to \infty \) limit (with \( R_+ = \ell \hat{r}_+ \) and \( r_- = r_0 \))
\[ ds^2 = \hat{r}_+^2 \left( 1 - \frac{r_0^2}{r^2} \right) \, dt^2 - \frac{r^2 \, dr^2}{\hat{r}_+^2 (r^2 - r_0^2)} + r^2 \left( d\varphi - \frac{\hat{r}_+ r_0}{r^2} \, dt \right)^2 \]
Flat space cosmologies (Cornalba & Costa ’02)

- Start with BTZ in AdS:
\[
ds^2 = -\frac{(r^2 - R_+^2)(r^2 - r_-^2)}{r^2 \ell^2} \, dt^2 + \frac{r^2 \ell^2 \, dr^2}{(r^2 - R_+^2)(r^2 - r_-^2)} + r^2 \left( d\varphi - \frac{R_+ r_-}{\ell r^2} \, dt \right)^2
\]

- Consider region between the two horizons \( r_- < r < R_+ \)
- Take the \( \ell \to \infty \) limit (with \( R_+ = \ell \hat{r}_+ \) and \( r_- = r_0 \))
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\]

- Go to Euclidean signature \( (t = i \tau_E, \hat{r}_+ = -i r_+) \)
\[
ds^2 = r_+^2 \left( 1 - \frac{r_0^2}{r^2} \right) \, d\tau_E^2 + \frac{r^2 \, dr^2}{r_+^2 (r^2 - r_0^2)} + r^2 \left( d\varphi - \frac{r_+ r_0}{r^2} \, d\tau_E \right)^2
\]

Note peculiarity: no conical singularity, but asymptotic conical defect!

Is FSC or HFS the preferred Euclidean saddle?

Question we want to address:

Daniel Grumiller — Flat space holography

Novel result: Cosmic phase transition
Flat space cosmologies (Cornalba & Costa ’02)

- Start with BTZ in AdS:

\[
\begin{align*}
 ds^2 &= -\frac{(r^2 - R_+^2)(r^2 - r_-^2)}{r^2\ell^2} \, dt^2 + \frac{r^2\ell^2 \, dr^2}{(r^2 - R_+^2)(r^2 - r_-^2)} + r^2 \left( d\varphi - \frac{R_+ r_-}{\ell r^2} \, dt \right)^2 \\
 ds^2 &= \hat{r}_+^2 \left( 1 - \frac{r_0^2}{r^2} \right) \, dt^2 - \frac{r^2 \, dr^2}{\hat{r}_+^2 (r^2 - r_0^2)} + r^2 \left( d\varphi - \frac{\hat{r}_+ r_0}{r^2} \, dt \right)^2 \\
 ds^2 &= r_+^2 \left( 1 - \frac{r_0^2}{r^2} \right) \, d\tau_E^2 + \frac{r^2 \, dr^2}{r_+^2 (r^2 - r_0^2)} + r^2 \left( d\varphi - \frac{r_+ r_0}{r^2} \, d\tau_E \right)^2 \\
\end{align*}
\]

- Consider region between the two horizons \( r_- < r < R_+ \)

- Take the \( \ell \to \infty \) limit (with \( R_+ = \ell \hat{r}_+ \) and \( r_- = r_0 \))

- Go to Euclidean signature (\( t = i\tau_E, \hat{r}_+ = -ir_+ \))

- Note peculiarity: no conical singularity, but asymptotic conical defect!
Flat space cosmologies (Cornalba & Costa ‘02)

- Start with BTZ in AdS:
  \[ ds^2 = -\frac{(r^2 - R_+^2)(r^2 - r_-^2)}{r^2 \ell^2} \, dt^2 + \frac{r^2 \ell^2 \, dr^2}{(r^2 - R_+^2)(r^2 - r_-^2)} + r^2 \left( d\varphi - \frac{R_+ r_-}{\ell r^2} \, dt \right)^2 \]

- Consider region between the two horizons \( r_- < r < R_+ \)
- Take the \( \ell \to \infty \) limit (with \( R_+ = \ell \hat{r}_+ \) and \( r_- = r_0 \))
  \[ ds^2 = \hat{r}_+^2 \left( 1 - \frac{r_0^2}{r^2} \right) \, dt^2 - \frac{r^2 \, dr^2}{\hat{r}_+^2 (r^2 - r_0^2)} + r^2 \left( d\varphi - \frac{\hat{r}_+ r_0}{r^2} \, dt \right)^2 \]

- Go to Euclidean signature \( (t = i \tau_E, \hat{r}_+ = -ir_+) \)
  \[ ds^2 = r_+^2 \left( 1 - \frac{r_0^2}{r^2} \right) \, d\tau_E^2 + \frac{r^2 \, dr^2}{r_+^2 (r^2 - r_0^2)} + r^2 \left( d\varphi - \frac{r_+ r_0}{r^2} \, d\tau_E \right)^2 \]

- Note peculiarity: no conical singularity, but asymptotic conical defect!

**Question we want to address:**

Is FSC or HFS the preferred Euclidean saddle?
Euclidean path integral

Evaluate Euclidean partition function in semi-classical limit

\[ Z(T, \Omega) = \int \mathcal{D}g \ e^{-\Gamma[g]} = \sum_{g_c} e^{-\Gamma[g_c(T, \Omega)]} \times Z_{\text{fluct.}} \]

boundary conditions specified by temperature \( T \) and angular velocity \( \Omega \)
Euclidean path integral

Evaluate Euclidean partition function in semi-classical limit

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boundary conditions specified by temperature \( T \) and angular velocity \( \Omega \)

Two Euclidean saddle points in same ensemble if

- same temperature \( T \) and angular velocity \( \Omega \)
- obey flat space boundary conditions
- solutions without conical singularities
Euclidean path integral

Evaluate Euclidean partition function in semi-classical limit

\[ Z(T, \Omega) = \int \mathcal{D}g \, e^{-\Gamma[g]} = \sum_{g_c} e^{-\Gamma[g_c(T, \Omega)]} \times Z_{\text{fluct.}} \]

boundary conditions specified by temperature \( T \) and angular velocity \( \Omega \)

Two Euclidean saddle points in same ensemble if

- same temperature \( T \) and angular velocity \( \Omega \)
- obey flat space boundary conditions
- solutions without conical singularities

HFS:

\[(\tau_E, \varphi) \sim (\tau_E + \beta, \varphi + \beta \Omega) \sim (\tau_E, \varphi + 2\pi)\]

FSC:

\[(\tau_E, \varphi) \sim (\tau_E + \beta, \varphi + \beta \Omega) \sim (\tau_E, \varphi + 2\pi)\]
Results

On-shell action (1/2 Gibbons–Hawking–York boundary term!):

\[ \Gamma = -\frac{1}{16\pi G_N} \int d^3x \sqrt{g} R - \frac{1}{16\pi G_N} \int d^2x \sqrt{\gamma} K \]
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Free energy:

\[ F_{\text{HFS}} = - \frac{1}{8G_N} \]
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- \( r_+ > 1 \): FSC dominant saddle
- \( r_+ < 1 \): HFS dominant saddle

Critical temperature:

\[ T_c = \frac{1}{2\pi r_0} = \frac{\Omega}{2\pi} \]
Discussion and generalization

- Free energy of FSC: \( F(T, \Omega) = -\frac{\pi T}{4G_N \Omega} \)
- Entropy: \( S = \frac{2\pi r_0}{4G_N} \) (BH area law)
- First law: \( dF = -S \, dT - J \, d\Omega \)
- Some unusual signs reminiscent of inner horizon black hole mechanics
- Critical temperature: self-dual point (w.r.t. flat-space “S-trafo”)

Generalization to TMG straightforward
Consistency with flat space chiral gravity Cardy formula:
\( S = \frac{2\pi}{\sqrt{\text{ch}}} = \frac{4\pi kr}{G_N} \)

Non-negative specific heat

Generalizations: should be easy to consider NMG, GMG, ... in 3D

Higher dimensions?
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Summary of the main result

Hot flat space

\[ ds^2 = dt^2 + dr^2 + r^2 \, d\varphi^2 \]

Flat space cosmology

\[ ds^2 = d\tau^2 + \frac{(E\tau)^2 \, dx^2}{1 + (E\tau)^2} + (1 + (E\tau)^2) \left( dy + \frac{(E\tau)^2}{1 + (E\tau)^2} \, dx \right)^2 \]

Flat space cosmology

\[ (\varphi \sim \varphi + 2\pi) \]

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Literature


Appendix: Coordinate transformation to Cornalba–Costa line-element

FSC in BTZ coordinates:

\[ ds^2 = \hat{r}_+^2 \left( 1 - \frac{r_0^2}{r^2} \right) \, dt^2 - \frac{r^2 \, dr^2}{\hat{r}_+^2 (r^2 - r_0^2)} + r^2 \left( d\varphi - \frac{\hat{r}_+ + r_0}{r^2} \, dt \right)^2 \]

Coordinate trafo:

\[ \hat{r}_+ t = -x \]
\[ r_0 \varphi = x + y \]
\[ (r/r_0)^2 = 1 + (E\tau)^2 \]
\[ E = \hat{r}_+/r_0 \]

FSC in CC coordinates:

\[ ds^2 = -d\tau^2 + \frac{(E\tau)^2 \, dx^2}{1 + (E\tau)^2} + (1 + (E\tau)^2) \left( dy + \frac{(E\tau)^2 \, dx}{1 + (E\tau)^2} \right)^2 \]

Daniel Grumiller — Flat space holography

Novel result: Cosmic phase transition